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Interaction of the Breaking and Non-Breaking Solitary Waves and Linear and Nonlinear Slopes Sea Bed by SPH Method

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ABSTRACT

Investigating solitary waves is critical for predicting the tsunami waves features in the coastal region. The Smooth Particle Hydrodynamic method (SPH) is the Lagrangian meshless method that is commonly used to simulate nonlinear waves and free surface problems. In this study, the SPH method is used to investigate the propagation of the breaking and non-breaking solitary waves over the linear and nonlinear sharp slopes sea bed. The presented SPH model is validated in comparing the experimental and another numerical model. The results show that the presented method prepares powerful tools to simulate the solitary wave propagation over the variable sea bed. Then, the transmitted solitary wave from the linear and nonlinear sharp slopes of the sea bed were compared. The results show that the solitary wave energy loss for waves propagating through the linear slopes is more than that of non-linear slopes. Moreover, the solitary wave transmission coefficient increases with increases in wave height in the linear and nonlinear slope. Also, the transmitted solitary wave height through the nonlinear slope is slightly more than the linear slope. Finally, the breaking Solitary waves when passing through the variable depth in the near shore with nonlinear slopes is investigated by comparing the wave free surface, streamlines, and orbital velocities before and after wave breaking. The results show that the solitary wave horizontal orbital velocity values after the wave breaking are increasing in comparison with before breaking.

1. Introduction

The tsunami is generated after the rapid scale-up turbulence of a volume of ocean water due to underwater disturbances like underwater earthquakes, landslides, and volcanic eruptions. The long waves with small amplitudes (usually less than 1 m of height and more than 100 kilometers of the wavelengths) are formed in the deep ocean water due to underwater disturbance. When this mass of water moves toward the coast, the waves height increases as the water depth decreases near the coast. These massive and dangerous long waves have caused many financial and fatality consequences in coastal regions. For example, the tsunami of 2004 in India with more than 230,000 dead, and the tsunami of 2011 in Japan with more than 20,000 dead and 360 billion dollars in financial consequences can be reviewed [1].

Since the surface profile of a tsunami is similar to that of solitary long waves, these waves have been widely used to simulate and study the physical behavior of tsunami waves in detail since the 1970s. In this regard, the solitary wave are used to study the tsunami run-up on the coastal slope (Di Risio et al., 2009 [2]; Knowles and Yeh, 2018 [3]; Ershadi, et al., 2019 [4]), the interaction between tsunami waves and structures (Fathi and Ketabdari, 2018 [5]; Ko and Yeh[6], 2019; Jensen, 2019 [7]; Vinodh and Tanaka, 2020 [8]; Wei et al., 2021 [9]; lin et al., 2023 [10]), and coastal sediment and morphology (Xiao et al., 2010 [11]; Li et al., 2019 [12]). It should be noted that there are some dissimilarities between the solitary and real tsunami waves in shallow water were noted by Madsen et al. 2008 [13], and Qu et al.[14].

When a solitary wave propagates in shallow water with a variable sea bed, several critical phenomena occur. Some of the solitary wave's energy is reflected, and some is lost due to friction on the seabed. Also, the remainder of wave energy pass over the seabed slope or underwater obstacles [Liu et al, 1991 [15]; Vaziri et al., 2011 [16]; Papoutsellis et al., 2018 [17]]. Moreover, as solitary waves pass over the variable seabed, small solitary waves are formed from the original wave, which is called the fission phenomenon. Understanding wave fission is important for predicting how waves will behave in shallow or variable-depth waters, which has implications for coastal engineering, marine navigation, and tsunami modeling. So far, many studies have been done on the solitary waves propagating through the variable bed some of which are mentioned below.

Seabra-Santon et al., 1987 investigated the interaction of solitary waves with a varying seabed using the finite difference method in conjunction with the shallow water equations. They compared the numerical results with experimental data [18]. Sriram et al. (2006) studied the same problem (Seabra-Santon et al. study) using the finite element method by solving the nonlinear potential theory [19]. Chowdhury and Sannasiraj (2013) simulated the propagation of solitary waves over both constant and varying seabeds using the SPH method [20]. They showed that the solitary waves split into several solitons when crossing the continental shelf. Tan et al., 2019 presented a numerical model to predict the number and amplitudes of the disintegrated solitons of solitary waves propagating over a step [21]. They also compared the numerical results with the experimental data from Seabra-Santon et al., 1987. Chao et al., 2021 prepared an experimental setup to study the interactions of solitary waves with a Submerged Step using the image-connection technique. They investigated the effect of changing the step height and solitary wave height of the transmission wave features [22]. Fang et al., 2022 studied the interaction of solitary waves and complex bathymetries with the different linear slopes [23]. They showed that two-layer Boussinesq model is a powerful tool for modeling the solitary wave propagation and transformation over the linear sea bed. Lin et al., 2023 prepared laboratory experiments to investigate the breaking of solitary waves propagating through submarine canyons [10]. They used a combination of particle image velocimetry and planar laser-induced fluorescence techniques to measure the free surface solitary wave data.

The extensive literature review presented above shows that so far no numerical or experimental studies have focused on the interaction of solitary waves with nonlinear slope variable seabed depth. In this study, the interaction of the generated solitary waves in NTW using the SPH method and the linear and nonlinear sea bed slopes. After explaining the theoretical formulation of the SPH method in Sec. 2, the presented SPH model is validated in comparing the experimental and another numerical model. Additionally, the solitary wave generated using the SPH method is validated by comparing it with the Boussinesq solitary wave equation. The results demonstrate that the presented SPH model accurately simulates solitary wave propagation through a variable seabed. After model validation, the interaction between the transmitted solitary wave and linear and nonlinear seabed slopes is investigated. The effect of the non-breaking solitary wave height (H) on the wave transmission coefficient (Ct) after propagating through the linear and nonlinear slopes is also studied. Finally, the phenomenon of solitary wave breaking on the nonlinear slope when passing through variable depth in the nearshore region is examined. Streamline and orbital velocities before and after wave breaking are also compared.

2. Smoothed Particle Hydrodynamics method

2.1. Kernel function

The basin equation in the Smoothed Particle Hydrodynamics method for each variable A(r) is shown as follows:

$$A(r) = \int A(r')W(r - r', h)dr' \qquad 1$$

Where, r and r' represent the location of the main and neighboring particles respectively, h is the effective radius of the neighboring particles, and $W(r - r', h)$ is the kernel function that is used to interpolate between the neighboring particles. Several equations are suggested for kernel functions. In this study, the cubic spline function was used, which is defined below:

$$W(r,h) = \begin{cases} \frac{10}{7\pi h^2}\left(1 - \frac{3}{2}q^2 - \frac{3}{4}q^3\right) & 0 < q < 1 \\ \frac{10}{28\pi h^2}(2-q)^3 & 1 < q < 2 \\ 0 & q > 2 \end{cases} \qquad 2$$

Where, q=r/h and $r = r_a - r'$ is the distance between particle a and its neighboring particles.

2.2. The conservation of the mass

The governing equations in the SPH method consist of the continuity and momentum equations as shown below [24]:

$$\frac{1}{\rho}\frac{\partial \rho}{\partial t} + \nabla . u = 0 \qquad 3$$

$$\frac{Du}{Dt} = -\frac{1}{\rho}\nabla P + g + \upsilon_0 \nabla^2 u + \frac{1}{\rho}\nabla \tau \qquad 4$$

Where ρ is the density, u is the velocity, g = (0, 0, -9.81) is the gravity acceleration, P is the pressure and υ_0 is the kinematic viscosity and τ is the SPS stress tensor.

The discretized form of the above equations as shown below [25] , [26]:

$$\frac{d\rho_a}{dt} = \rho_a \sum_b \frac{m_b}{\rho_b} u_{ab}.\nabla W_{ab} \qquad 5$$

$$\frac{du_a}{dt} = g + \sum_b m_b\left(\frac{P_a}{{\rho_a}^2} + \frac{P_b}{{\rho_b}^2} + \frac{\tau_a}{{\rho_a}^2} + \frac{\tau_b}{{\rho_b}^2}\right) + \sum_b m_b \frac{4\upsilon_0 r \nabla_a W_{ab}}{(r_a + r_b)(|r_{ab}|^2 + \eta^2)} \qquad 6$$

where, $\eta^2 = 0.01 * h^2$, $u_{ab} = u_a - u_b$ is the velocity difference between the interpolation particle a and neighboring particle.

2.3. Equation of State

In this study, the fluid is considered weakly compressible. Therefore, the equation of state (EoS) is used to compute the pressure term ([27]).

$$P = B\left[\left(\frac{P}{\rho_0}\right)^{\gamma} - 1\right] \quad 7$$

$$B = \frac{C_0 \rho_0}{\gamma} \quad 8$$

Where B is the constant parameter which is related to Bulk module, $\gamma = 7$ is the polytropic constant and C_0 is the sound velocity in the reference density. It should be noted that c_0 is not the real speed of sound and this value is restricted to be at least 10 and maximum 40 times the maximum fluid value to ensure that the density variations are less than 1%.

2.4. XSPH Method

The Particles movement is modified by monaghan according to the following equation [28].

$$\frac{\partial r_a}{\partial t} = V_a + \varepsilon \sum_b \frac{m_b}{\rho_{ab}} V_{ab} W_{ab} \quad 9$$

In which, ρ_{ab} and u_{ab} is the density and the average velocity between a and b. also, ε is a constant parameter which is located between 0 and 1 and is considered 0.5.

2.5. Solitary wave generation

The solitary wave free surface is defined based on long wave theory on position x and time *t* as following:

$$\eta(x,t) = H sech^2\left[\sqrt{\frac{3H}{4d}}\,(x - ct)\right] \quad 10$$

$$c = \sqrt{g(H_0 + d_0)} \quad 11$$

Where, C is the solitary wave velocity, H is wave height, and d_0 is water depth.

The piston type wave-maker is used to generate a solitary wave in shallow water according to bellow equation [29].

$$x(t) = \frac{2H}{d_0\beta}\,\frac{d_0 \tanh\left(\frac{\beta c(t-t_0)}{2}\right)}{d_0 + h\left(1 - \tanh^2\left(\frac{\beta c(t-t_0)}{2}\right)\right)} \quad 12$$

$$\beta = 2\sqrt{\frac{3H^3}{4{d_0}^2(H + d_0)}} \quad 13$$

where, x is the position of the piston type wave maker at different times and β is the coefficient of boundary lag. Also, the speed of the wave paddle is obtained from the below equation:

$$u_{x,t} = \frac{\Delta x}{\Delta t} \quad 14$$

It should be noted that the Wave Maker is modeled with zero speed and displacement particles in the SPH method.

3. Simulation of solitary wave in constant depth

In this section, the generated solitary wave is validated by the analytical Boussinesq equation. In this regard, the simulated wave flume in the SPH method is shown in Fig. 1.

Fig. 1: The simulated wave flume in the SPH method to validate solitary wave generation

The simulated solitary wave free surface at locations, x = 5 and x = 8 is presented in Fig. 2. Also, the percentage difference between the SPH simulation and the Boussinesq equation are shown in Table 1. These results demonstrate that the solitary wave simulated using the SPH method shows good agreement with the Boussinesq equation. Furthermore, Wave velocity based on the Boussinesq equation (Eq. (11)) and SPH method (C=Δx/Δt) are compared. The results indicate that the Wave velocity in Boussinesq models is equal to 1.469 m/s and in the SPH method is 1.473. Therefore, the proposed SPH method demonstrates sufficient accuracy in comparison with the Boussinesq theory.

Fig. 2: Validation of the generated solitary waves by SPH method in comparing Boussinesq models in, a: x = 5 m, b: x = 8 m.

3.2. Solitary wave validation when propagate through the sudden change in bed

When the solitary wave propagates in shallow water with a variable bed, some parts of the solitary wave energy are lost due to reflection and friction on the sea bed. Additionally, as the solitary waves pass over the variable bed, small solitary waves are formed behind the original wave, a phenomenon known as fission. The fission can affect wave energy distribution, which is crucial for designing coastal structures, predicting wave impact, and managing coastal erosion. Understanding how waves break up into smaller waves can help in predicting the behavior of tsunamis and storm surges as they approach shorelines with varying depths. In this section, the simulated solitary wave crossing the variable bed by the presented SPH method is validated against experimental data (Seabra santon et al, 1987 [18]) and compared with other SPH numerical results (Choudhary et al., 2013 [20]). The parameters used in the current SPH model are shown in Table 1.

Table 1: the Parameters used in the presented SPH simulating

Variable Description	Input data
Kernel	Cubic Spline
Time-Stepping Algorithm	Simplistic
Viscosity Treatment	Laminar+SPS
Equation of State	Tait
Box dimension (m)	20, 0.6
Particle spacing (m)	0.005, 0.005
CFL number	0.2

sound speed coefficient	20

The experimental wave flume used by Seabra Santon et al, with a length of 16 m, an initial water depth of 20 cm, a secondary water depth of 10 cm, and a solitary wave height of 3.65 cm, is shown in Fig. 3. The numerical wave set up by the presented SPH method is shown in Fig. 4. As depicted in this figure, several small solitary waves are formed as the main solitary wave propagates over the variable bed. Also, the solitary wave free surface generated by the presented SPH method is compared with the experimental results by Seabra Santon et al. (1987) and numerical SPH results by Choudhary et al. (2013) in Fig. 5. This comparison demonstrates that the presented SPH model achieves high accuracy in simulating solitary waves crossing a variable bed.

Fig. 3: The experimental wave flume used by Seabra santon et al.

Fig. 4: Simulating the solitary waves passing the variable bed using the presented SPH method

Fig. 5: comparing the presented results with the experimental results by Seabra Santon et al. (1987) and numerical SPH results by Choudhary et al. (2013)

4. Comparison of the solitary wave propagation through the curve and linear slope

In this section, the propagation of solitary waves over linear and nonlinear variable sea beds is compared. The simulated solitary waves and the corresponding wave flumes with linear and nonlinear slopes are illustrated in Fig. 6. The wave flumes have a total length of 10 m, with an initial water depth of 20 cm, a secondary water depth of 10 cm, and a solitary wave height of 4.5 cm.

Fig. 6: The simulated solitary wave passing through a: linear slope, b: nonlinear slope

The solitary wave free surface while passing over the linear and nonlinear slope beds is compared in Fig. 7. This figure indicates that the area below the passing solitary wave free surface over the nonlinear slope is slightly larger than that of the linear slope. Consequently, it can be concluded that wave energy loss for waves propagating over linear slopes is greater than for those propagating over nonlinear slopes. The orbital velocities in X direction at the solitary wave crest in X=2.5m and X=3m for the linear and nonlinear slops are compared in Fig. 8. It is observed that the orbital velocities at the solitary wave crest while passing over linear slopes are higher, indicating a greater likelihood of wave breaking on linear slopes compared to nonlinear slopes. The reason for this is that the wave's shoaling process is less abrupt compared to linear slopes. The depth changes more gradually, allowing the wave to adjust smoothly. Consequently, the increase in orbital velocities near the crest is less pronounced, as the wave's energy is distributed over a larger area. As a result, the likelihood of wave breaking is reduced compared to linear slopes

Fig. 7: Comparison the solitary wave free surface when passing through the linear and nonlinear slope

Fig. 8: The orbital velocities in X direction at the solitary wave crest a: X=2.5m, b: X=3m

4.1. The solitary wave transmission coefficient

In this section, the effect of the solitary waves height (H) on the wave transmission coefficient (Ct) after passing through the linear and nonlinear slops is investigated. Figure 9 and Table 2 illustrate the relationship between solitary wave height and the transmission coefficient. These results indicate that the transmission coefficient increases with increasing wave height for both linear and nonlinear slopes. This finding suggests that the impact of water depth variations on incident solitary waves diminishes as wave height increases. Additionally, Figure 9 demonstrates that the transmission coefficient for solitary waves passing over nonlinear slopes is slightly higher than for linear slopes. The interaction of the wave with a nonlinear slope can create complex boundary layer effects that might reduce energy dissipation compared to a linear slope. This reduction in dissipation can lead to a higher transmission coefficient. While, linear slopes might have simpler boundary layer interactions that result in slightly higher energy loss.

Table 2: The solitary wave effect on the transmission coefficient

d1 (cm)	H (m)	Ct for Linear slope	Ct for Nonlinear slope
20	6	.862	.874
20	8	.881	.892
20	9.5	.907	.922

Fig. 9: Comparison of the transmission coefficient of passing solitary wave through the linear and nonlinear slope

4.2. Breaking Solitary Wave with Changing Water Depth

Solitary waves break when the water depth decreases significantly due to changes in sea bed level. its study is essential for advancing our understanding of coastal processes, improving hazard prediction and mitigation, and designing sustainable coastal infrastructure. In this section, the phenomenon of breaking solitary waves on a nonlinear slope while passing through variable depths near the shore is investigated.

The wave flume used to simulate solitary wave breaking near the shore has a length of 10 m, an initial water depth of 50 cm, a secondary water depth of 10 cm, and a solitary wave height of 15 cm, as shown in Fig. 10. When the solitary wave propagates over steep slopes, parts of its energy are lost due to reflection, fission, and friction, while the remaining energy is dissipated during wave breaking. The solitary wave free surface both before and after breaking is shown in Fig. 11.

Fig. 10: The simulated wave flume in SPH model

Fig. 11: The solitary wave free surface before and after wave breaking

To investigate the wave-breaking phenomenon more precisely, the streamlines and orbital velocities in the X and Y directions are studied before and after the breaking. In this regard, the streamlines before and after the breaking are compared in Fig. 12. It should be noted that the angle of the streamline arrow with horizontal axis $\alpha = \tan^{-1} u/v$ in where u and v are horizontal and vertical orbital velocities. The orbital velocities in the X and Y directions, studied before and after the break, are compared in Fig. 13 and Fig. 14, respectively. Figure 13 shows that the horizontal velocity at the solitary wave crest is at a maximum. Additionally, the horizontal velocity values of the solitary wave increase after wave breaking compared to before the break. It is due to energy redistribution, momentum transfer, wave transformation, and nonlinear interactions.

Fig. 12: The streamline solitary waves, a before wave breaking, b: after wave breaking

Fig. 13: The solitary waves orbital velocity in x direction, a: before wave breaking, b: after wave breaking

Fig. 14: The solitary waves orbital velocity in x direction, a: before wave breaking, b: after wave breaking

5. Conclusion

In this study, solitary waves were generated in a Numerical Tank Wave (NTW) using the SPH method to investigate their interaction with linear and nonlinear slope sea beds. The presented SPH model was validated by comparing the results with experimental data and other numerical models. Additionally, the generated solitary wave using the SPH method was validated by comparing it with the Boussinesq solitary wave equation. The results demonstrated that the presented method accurately simulates solitary waves propagating through variable sea beds. Subsequently, the transmitted solitary waves from linear and nonlinear sea bed slopes were compared. The effect of solitary wave height (H) on the wave transmission coefficient (Ct) after propagating through linear and nonlinear slopes was also investigated. Finally, the phenomenon of breaking solitary waves on nonlinear slopes, while passing through variable depths near the shore, was examined. Also, the streamlines and orbital velocities before and after solitary wave breaking were compared. The main results of this study are summarized as follows:

1) The solitary wave energy loss for waves propagating through linear slopes is greater than (about 3%) that for nonlinear slopes.
2) Since the orbital velocities at the solitary wave crest passing through the linear slopes are greater, the possibility of waves breaking on linear slopes is greater than the nonlinear slopes.
3) The transmission coefficient increases with increasing wave height in both linear and nonlinear slopes.
4) The transmission coefficient for solitary waves passing through nonlinear slopes is slightly higher (about 5%) than that for linear slopes.
5) The solitary wave horizontal velocity values after wave breaking increase (more than 60 %) compared to those before breaking.

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