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An Approximate Simple Computational Method to Determine Hydrodynamic Forces on Drag Dominated Offshore Structures

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ABSTRACT

Offshore structures are subjected to highly dynamic, irregular external loads from the air, water and soil. One of the most important forces affecting design, construction, installation, and maintenance of such structures considerably is wave force. Accordingly, accurate calculation of the wave force acting on offshore structures is one of the first steps of design process to ensure structural safety of all components under different environmental conditions. Nevertheless, such calculations are not only difficult but also computationally expensive. The main reasons for such complexities are the probabilistic and randomness of sea waves as well as the dynamics of Fluid Structure Interaction (FSI) which make the assessment of wave loading on offshore structures a challenging procedure in offshore engineering practice. It is therefore the main objective of this research to present effective alternative approximation methods based on the Morison's equation for determining wave loads acting on the structural components of the offshore supporting structures that are slender compared to the wave length (e.g. legs, braces, spokes and mooring lines). The comparison results show that the proposed method can be more efficient with time and also a reliable method in initial design of offshore platforms with acceptable results compared to the conventional approach.

1. Introduction

Offshore structures are subjected to different environmental conditions with complex loading combinations such as waves, currents, wind, and sometimes earthquakes or ice [1-3]. Among these forces, the wave-induced loadings are often the most important dynamic excitations for most offshore structures. Therefore, accurate calculation of wave forces acting on offshore structures is essential for safe and reliable design in marine engineering. However, the nonlinearity of the wave loadings would be the main difficulty in such computations, even if the irregularity of wave and nonlinearities of the structure are ignored. Accordingly, a time domain analysis is required in order to determine the most accurate response especially for dynamically sensitive structures, e.g., drag dominated structures. Nevertheless, such calculations are conceptually complicated and computationally expensive. Similarly, experimental methods and local measurements as the other techniques for structure response calculations and behaviour evaluation are not only costly but also very difficult to perform. It is therefore practically necessary to develop worthwhile simplified models with shorter analysis time duration than usual models for engineering analysis applications at an early stage of the design process. The importance of establishing such models is that it does not only allow the engineers to have a better interpretation of the dynamic behaviour of the structure by driving analytical solution but also provides a good basis for validation of numerical method's results [4].

Numerous studies have been carried out using Morison's equation [5] for wave loading estimation on the cylindrical slender structural members of offshore structures. However, despite the good estimation of wave loads provided by this model, the nonlinearity existing in its drag force component makes the dynamic analysis of the drag dominated structures prohibitively time consuming and therefore it has limited applications in usual design and assessment practice of offshore platforms. Consequently, developing more economic and reliable analytical solution techniques are desirable.

Over the last few decades, many research studies have been dedicated to modify and linearize Morison's model in order to simplify the load and response estimation procedure of offshore structures comprised of cylindrical slender elements. The most extensively used linearized form of Morison's equation was proposed by [6-8] using equivalent linearization technique developed by [9]. The main advantage of this linearization besides drastically reduction of the computational time is that the excitation force and corresponding response become stochastically stationary and Gaussian [10, 11], so many efficient numerical methods can be used to assess the structural behaviour in frequency domain [12, 13]. Nevertheless, since this approximation significantly underestimates the distribution of wave force [14, 15]; [16] modified the original linear form of Morison's equation and obtained a new coefficient for the linearized drag force component, in which it gives a better estimation of the nonlinear wave-induced forces. Also, [4] derived a new linearized form of drag component utilizing a same technique with considering the probability density instead of spectrum of the wave force used to drive the original model which provides a more accurate approximation of wave force than the Borgman's model. Moreover, [17] discussed the major limitation with the classical Gaussian equivalent linearization method using the well-known mean square error criterion and proposed an approximate series including the conventional one as the first approximation for the coefficients of the equivalent linearized drag force in the Morison's equation through a new mean square criterion based on Hermite polynomial error sample functions. The Borgman's linearized model was developed for unidirectional wave forces; therefore, due to limitations of the model with vector operation rules, [18] added some modification and presented a new linearized model applicable for multi-directional wave flow. [19] extended the conventional linearized model using statistical linearization technique for nonzero-mean stochastic in case of waves and currents co-occurrence. [20] discussed the great limitations of Borgman's linearization technique in extreme wave loading and wave-induced fatigue loading calculations illustrated well by [21] and used an alternative linearization to make this equation applicable for such cases. Apart from the aforementioned studies on linear predictions, other higher-order solutions have also been proposed in order to reduce the errors originated by the linearization approaches as much as possible [21].

The main objective of this study is to introduce new simplified approximation models for hydro dynamical wave loads calculation applied on cylindrical members of offshore platforms based on linear wave theory and the Morison's equation. For this purpose, first, the non-linearized and linearized model of Morison's equation for regular wave cases with and without considering the effects of the instantaneous free water-wave surface are discussed. Then, the alternative simplified methods based on two ideas: linear and uniform distributions of wave force in depth, are developed. Afterwards, in order to verify the applicability and accuracy of the proposed calculation methods, the important parameters governing the behavior of the offshore structures (i.e., wave-induced base shear and overturning moment) for a single cantilever representative cylinder in a unidirectional flow are compared with the nonlinear and linearized Morison's wave-induced forces.

2. Wave Loading

The evaluation of structural and motion responses are essential in the assessment of operability of offshore structures. The design loads especially hydrodynamic forces have crucial roles in the design of offshore structures. Therefore, many research efforts have been dedicated on the development of an appropriate hydrodynamic load calculation modeling procedure especially the computation of wave induced loads for either the fatigue or extreme cases. In the recent years, there has been a significant improvement in the prediction of wave loads imposed on offshore structures. In this regard, linear and nonlinear methods including time domain numerical approaches and frequency domain techniques using different wave theories (linear and non-linear) and wave loading calculation methods (i.e., Morison's method, Diffraction theory, Froude-Krylov method, and Computational Fluid Dynamic (CFD)) as well as probability analysis have been utilized [22].

3. Morison Equation Considering Deep Water-Wave Conditions

The correct calculation of the wave load on the support structures depends on selecting an appropriate load-calculation model in the simulation (see Figures 1 and 2). The suitable approach to calculate the total time-varying wave loading per unit length acting normal to the slender members (i.e., the ratio of member diameter to the wavelength is less than 0.2) is Morison's equation. According to this theory, such structures do not significantly affect the wave particle motions and the wave induced forces originate from inertia and viscous effects. The horizontal wave loads (i.e., Morison's forces) per unit length for certain depth of motionless member below the mean water surface can then be expressed as the sum of inertia force (i.e., added mass), f_I, and drag force, f_D, as follows [23-25].

$$f(z,t) = f_I + f_D = \rho C_I V_b a_x + \frac{1}{2}\rho C_D A_p u_x |u_x| \tag{1}$$

In which, ρ is the water density, C_I is the hydrodynamic inertia coefficient, C_D is the hydrodynamic drag coefficient. Inertia and drag coefficients are functions of the Reynolds number (R_e), Keulegan-Carpenter number (K_c) and relative surface roughness of structural member [26, 27]. Values of these coefficients can be obtained from lab or field model tests; however, for conventional engineering works, the values of these coefficients are assumed to be practically constant. According to the [28], the hydrodynamic drag and inertia coefficients are in the range of 0.6 to 1 and 1.5 to 2, respectively. Also, V_b is the volume of the element, A_p is the projected area of the member normal to the direction of the wave motion field, and a_x and u_x are the horizontal wave-induced acceleration and velocity components of the water particle normal to the axis of the member, respectively. The wave particle velocity and acceleration can be computed using different wave theories developed for different specific range of depths; however, the linear small-amplitude wave theory (i.e., Airy) normally provide promising and reliable estimations of this parameter with a shorter computational time, which is a very important factor in real structural design practices. According to the small-amplitude theory as a derivation of first-order velocity potential theory, the horizontal component of water particle kinematics can be determined as follows:

$$u_x = \frac{\omega H}{2}\left(\frac{\cosh\left[k\left(z+d\right)\right]}{\sinh kd}\right)\cos\left(kx-\omega t\right) \tag{2}$$

$$a_x = \frac{\omega^2 H}{2}\left(\frac{\cosh\left[k\left(z+d\right)\right]}{\sinh kd}\right)\sin\left(kx-\omega t\right) \tag{3}$$

where, H is the wave height, T is the wave period, d is the water depth, z is the depth of member below the mean water surface, and $k=2\pi/\lambda(wave\ length)$ and $\omega=2\pi/T$ are the wave number and wave angular frequency, respectively. Hence, by substituting the horizontal component of water particle kinematics given by Eq. (3) into Eq. (2) and integrating the force per unit length acting on the structure from the mud line up to the mean water surface level (see Figure 3), the total wave force can be calculated as follows:

$$F=\int_{-d}^{0} f\ (z,t)dz \tag{4}$$

$$F_I = -\frac{\rho C_I}{2k}\frac{\pi D^2}{4}\omega^2 H sin(\omega t) \tag{5}$$

$$F_D = \frac{\rho C_D D}{32k}\left(\omega H\right)^2 \times \left(\frac{2kd+sinh(2kd)}{sinh^2(kd)}\right)cos\,\omega t\,\left|cos\,\omega t\right| \tag{6}$$

This equation (i.e., Eq. (4)) would be limited only to drag force component in cases where the structures are drag dominated, $D/H<0.16$ (see Figure 2). This can be shown as:

$$\frac{F_{D_{\max}}}{F_{I_{\max}}} = \frac{1}{4\pi}\frac{C_D}{C_I}\left(\frac{2kd+sinh(2kd)}{sinh^2(kd)}\right)\left(\frac{H}{D}\right) \tag{7}$$

$$Deep\,water \rightarrow sinh(kd) \approx \frac{e^{kd}}{2}$$

$$\left(\frac{2kd+\frac{e^{2kd}}{2}}{\frac{e^{2kd}}{4}}\right)_{d\rightarrow\infty} \approx \left(2+\frac{8kd}{e^{2kd}}\right)_{d\rightarrow\infty} \approx 2 \tag{8}$$

$$C_I=1\,,\,C_D=1$$

$$\frac{F_{D_{\max}}}{F_{I_{\max}}} = \frac{1}{2\pi}\left(\frac{H}{D}\right) \tag{9}$$

$$\frac{D}{H} << \frac{1}{2\pi} \rightarrow \frac{F_{D_{\max}}}{F_{I_{\max}}} >> 1$$

Therefore, the total wave loads in such cases, which encompasses the kind of structures are considered in this study, can be rewritten as follows:

$$f\ (z,t) = f_D = \frac{1}{2}\rho C_D A_p u_x\left|u_x\right| \tag{10}$$

The desribe kinematics of the wave in the Airy linear wave theory is only valid up to the mean sea level; Accordingly, to provide a more realistic representation of wave loads near the mean sea level, it is neccessary to apply some modifications to the water particle velocity definition obtained from the linear wave theory in order to take into account the effects of free surface fluctuations. For this purpose, various extrapolation teqniquies such as Wheeler, vertical, and linear stretching methods, which have been widely utlized in practice, can be used. İn the vertical and linear stretching methods, the water particle velocity at the mean sea level is assumed to be extrapolated to above MSL with zero and constant partial derivation, respectively. In the Wheeler stretching method, which is used in this study as the most commonly utilized method, the extrapolation is applied through coordinate mapping as follows, so that the profile is stretched and redistributed to instantaneous free surface elevation of wave [29]:

$$z = \frac{z_s - \eta}{1+\frac{\eta}{d}} \tag{11}$$

In this case, the wave kinematics and total wave force can be calculated as follows:

$$u_x = \frac{\omega H}{2} \times \frac{cosh\left[k\left(z+d\right)\left(\frac{d}{d+\eta}\right)\right]}{sinhkd}cos(kx-\omega t) \tag{12}$$

$$a_x = \frac{\omega^2 H}{2} \times \frac{\cosh\left[k\left(z+d\right)\left(\frac{d}{d+\eta}\right)\right]}{\sinh kd}\sin(kx-\omega t) \quad (13)$$

which yields to:

$$F^{Stretched} = \int_{-d}^{\eta} f\,(z,t)dz \quad (14)$$

$$F_I^{Stretched} = \left(\frac{d+\eta}{d}\right)F_I \quad (15)$$

$$F_D^{Stretched} = \left(\frac{d+\eta}{d}\right)F_D \quad (16)$$

It should be noted that in cases where the structural movement is considerable, the relative velocity and acceleration contributed from the fluid-structure interaction must be accounted in the calculation of hydrodynamic forces.

For the ultimate/collapse capacity design purposes, the global analysis of structures through parameters giving the dominant responses is required [30]. Such evaluation is usually performed by calculating the maximum global loads (i.e., base shear and overturning moment) which may govern the design of braces and leg members. To obtain the maximum base shear and overturning moment, since each member will not be attaining the maximum loads due to its location relative to wave; hypothetically, it should be assumed that the wave crest is positioned relatively at the origin of each member even though the obtained responses and hence the design would be highly conservation [31, 32]. These parameters can be calculated for as follows:

$$M_T = \sum_{i=1}^{N}\left[F_i * \Delta z_i * (d - |z_i|)\right] \quad (17)$$

$$F_T = \sum_{i=1}^{N}\left[F_i * \Delta z_i)\right] \quad (18)$$

where N is the number of segments along the length of element, F_i is the lateral wave forces per unit length at the centroid of segment i, Δz_i is the subdivision steps size of the member length associated with node i, d the depth of water, and z_i is the depth of node i below the mean water surface.

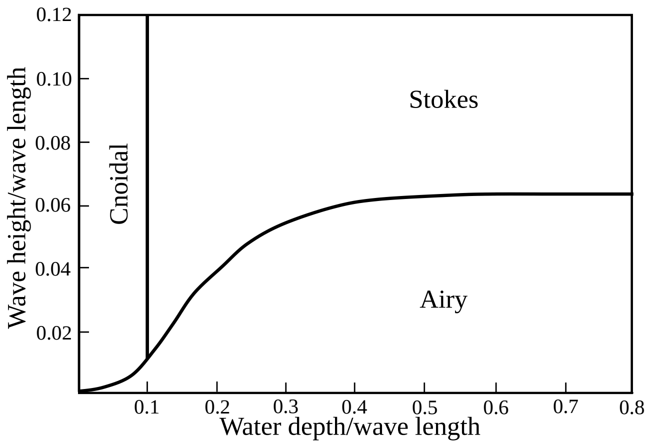


Figure 1. The validity ranges of different water wave theories [33].

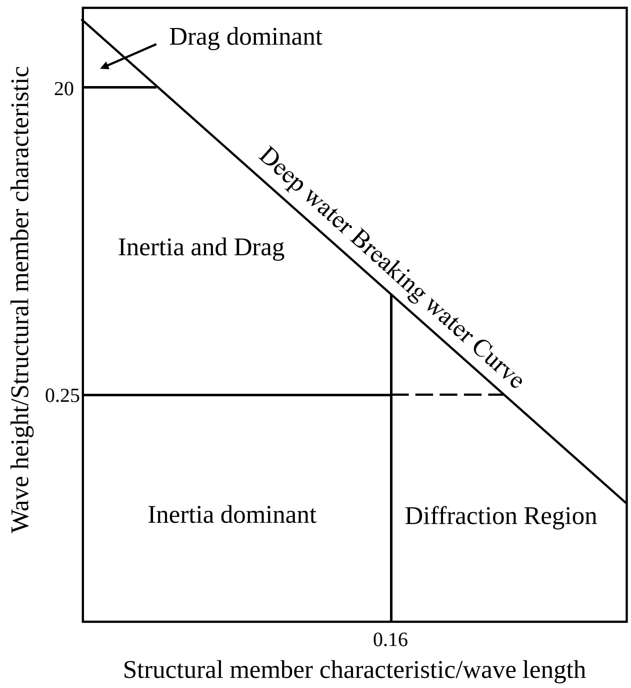


Figure 2. The dominating wave force regimes [23].

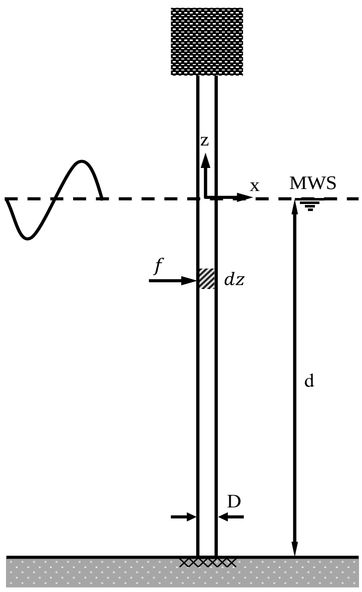


Figure 3. The schematic of the partial force of the wave acting on a representative contilevered vertical cylinder

4. Linearization of Morison's Equation

In the Morison's equation, the inertia force term is a linear function of particle acceleration components while the drag force term is a nonlinear function of particle velocity components which makes the implementation of wave loading calculation algorithm conceptually and practically very difficult. As a result, linearization of this term is very important in cases like that is intended to use a mixed approach and combining the Morison's equation with the diffraction potential theory in order to solve the governing equation of motion in the frequency domain. Consequently, in regular waves where a cosine variation in velocity, $u = u_0 \cos \omega t$ is assumed, the nonlinear drag term of Morison's equation can be linearized using the Fourier series expansion as follows:

$$F_D = A_D \cos \omega t . |\cos \omega t| = A_D \left(a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega t + b_n \sin n\omega t \right) \right) \tag{19}$$

In which,

$$\begin{aligned} a_0 &= \frac{\omega}{2\pi} \times \int_0^{2\pi/\omega} A_D \cos \omega t \, |\cos \omega t| \, dt = 0 \\ a_n &= \frac{\omega}{\pi} \times \int_0^{2\pi/\omega} A_D \cos \omega t \, |\cos \omega t| \cos \omega t dt = A_D \times \left(\frac{8}{3\pi} \cos \omega t - \sum_{n=1}^{\infty} \frac{8 \sin\left((2n+1) \omega t \right)}{\pi (2n-1)(2n+1)(2n+3)} \right) \\ b_n &= \frac{\omega}{2\pi} \times \int_0^{2\pi/\omega} A_D \cos \omega t \, |\cos \omega t| \sin \omega t dt = 0 \end{aligned} \tag{20}$$

The higher-order harmonics have less effects on the response due to less amplitude and energy than the fundamental harmonic and structural dynamics [34]. Therefore, the fundamental harmonic seems to be sufficient order of expansion and the linearized drag force can be given by:

$$F_D = A_D \left(\frac{8}{3\pi} \cos \omega t + higher\ \ harmonics \right) \approx A_D \frac{8}{3\pi} \cos \omega t \tag{21}$$

5. Equivalent Simplified Approximation Expressions of Morison's Equation

Another approach to reduce the complexity and computational cost of using drag term directly into the numerical wave loading calculation algorithm in situations mentioned before especially cases involved with high nonlinearities is to develop much simpler yet precise mathematical formulas to estimate the wave-induced forces. Accordingly, in this study, to overcome this problem, first, the usefulness of deep water limitation which is a reasonable assumption in the procedure of driving the simplified model for calculating the wave forces acting on Morison's type framed structures is considered. Based on this limitation, the terms in bracket in Eqs. (2) and (12), which show the variations of components over the vertical water column, can be rewritten as follows:

$$Deep\,water: \left\{ \begin{array}{l} \left(\dfrac{cosh\left[k\left(z+d \right) \right]}{sinhkd} \right) \\ \left(\dfrac{cosh\left[k\left(z+d \right) \left(\dfrac{d}{d+\eta} \right) \right]}{sinhkd} \right) \end{array} \right\}_{d \to \infty} = e^{kz} \tag{22}$$

In which, substitution of this simplification into the horizontal component of water particle velocity given by Eq. (2) yields to the following equation for wave force per unit length on drag dominated structures:

$$f(z,t) = \frac{1}{2} \rho C_D D \left(\frac{\omega H}{2} \right)^2 \times e^{2kz} cos\left(kx - \omega t \right) \left| cos\left(kx - \omega t \right) \right| \tag{23}$$

By taking into account the unit values with appropriate order of magnitude for parameters including the water density, $\rho = 1000 kg / m^3$, diameter of the member, $D = 1\,m$, and hydrodynamic drag coefficient, $C_D = 1$, the magnitude of maximum total unit wave load at the mean water surface level, f_0, in a sea state can be obtained as follows:

$$f(z,t) = f_0 e^{2kz} cos\left(\omega t \right) \left| cos\left(\omega t \right) \right| \tag{24}$$

where,

$$f_0^{[kg]} = 50 v_0^2 \;\;,\; v_0^2 = \left(\frac{\omega H}{2} \right)^2 \tag{25}$$

The reason for deriving this equation is to show that the magnitude of maximum total unit wave force at the mean water surface level in drag dominated structure can be simply carried out by knowing only the value of horizontal component of water particle velocity. This outcome is very important because it can be very usful in practical cases. Accordingly, for further simplification, simplified expressions for term, v_0, the amplitudes of horizontal water particles velocity at the mean sea level are elaborately extracted using the curve fitting teqnique through least-squares regressive minimization function. This approach then leads to concise expressions of wave kinematics within a good range of accuracy which at last results in very practical, reliable, and straightforward wave calculation models for engineering applications in deep water wave conditions. For this porpuse, the relationship between wave height and wave period, $T = 2.94\sqrt{H}$ [29] and dispersion equation definition for deep water conditions, $\omega^2 = gk$ [35] are first considered. Then, using linear regression on these terms yield to the following expressions: (1) $H \approx 1.8T - 6.2$ and (2) $H/\lambda = 0.07$. Figure 4a and b shows the derived linearized relationship between wave height, wave period, and wavelength for deep water conditions. Subsituation of achieved extracion into Eq. (10), the horizontal component of the water particle velocity can then be calculated in a simpler way for waves in deep water conditions, in which the wave height/period is the only parameter needed. Figure 5 shows the water particle velocity as a function of wave height and period using the aforementioned linearized relationships.

$$v_0 = 0.25H + 1 \qquad (26)$$

$$v_0 = 0.5T - 1 \qquad (27)$$

İt can now be cleary seen that by knowing only one of the main parameters of the wave (i.e., height/period) in a sea state, the magnitude of maximum total unit wave force at the mean water surface level can easily be obtained and to calculate the actual magnitude it just needed to multiply the output by the real values for the water density, diameter of the member, and hydrodynamic drag coefficient.

At last, to calculate the total unit wave-induced base shear force and overturning moment, the distribution of wave forces along each member at any moment is also required. Considering the location of platform members at different depths relative to the wave, each member attains a portion of induced loads. Accordingly, in the present study, the definition of wave penetration depth (Eq. (28)), which seems to play an important role in the amount of forces exerted to the structure, is used to develop two simple approximate methods, in which only one of the main characteristics of the wave in a sea state (i.e., significant wave height/spectral peak frequency) is needed to calculate the base shear and overturning moment of the maximum wave force on the structure.

$$PD(z) = e^{2kz} \qquad (28)$$

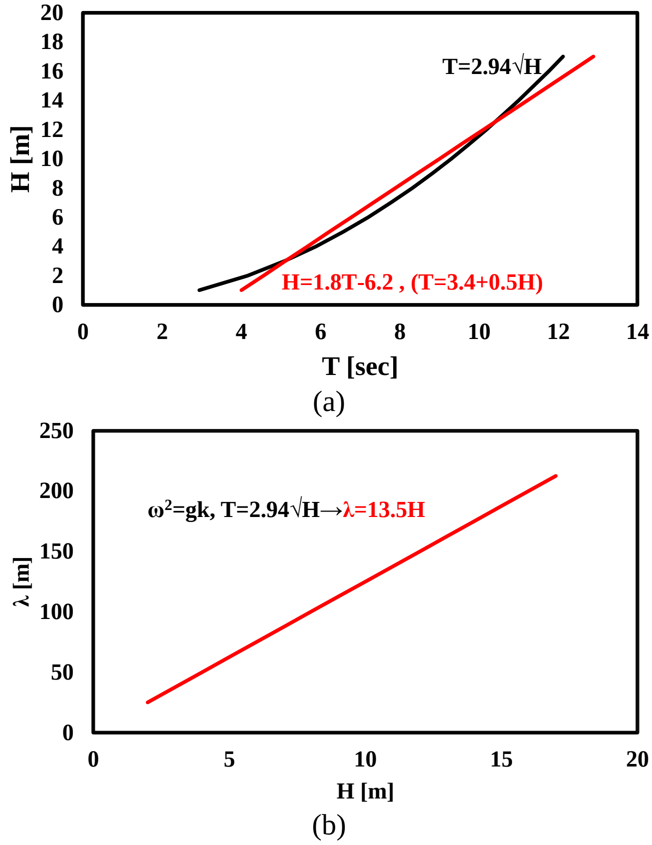


Figure 4. **The linearization diagrams of the relationships between (a) the wave height and period (b) the wave height and length.**

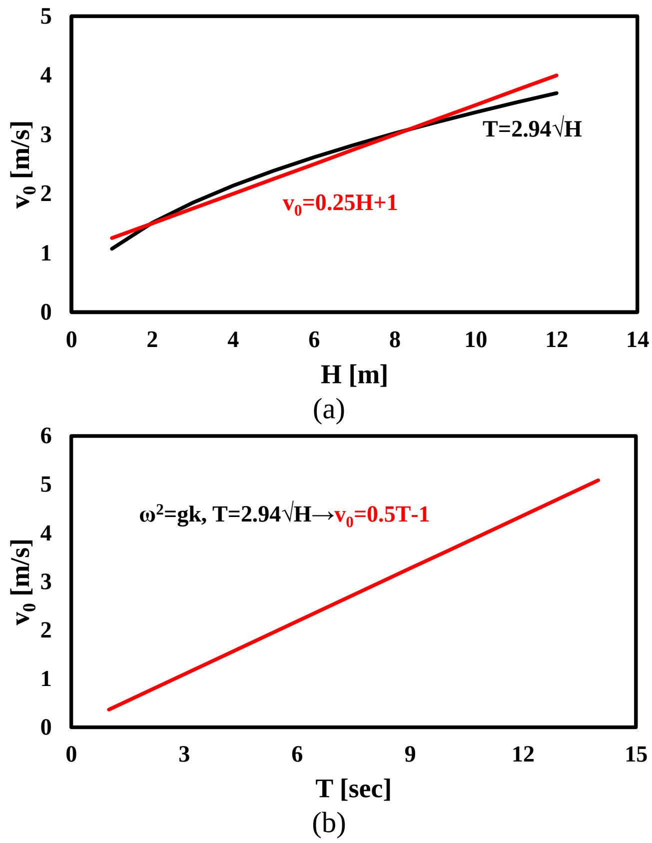


Figure 5. **The linearization diagrams of the relationships between (a) the wave height (b) the wave period and horizontal component of the water particle velocity.**

In the first method, the exponential distribution of the wave force in depth is replaced by a linear varying distributed (LVD) load segment along the projected length of the members normal to the direction of wave propagation at the instant of maximum wave force. This method is developed based on being able to calculate the total wave force on each member with 90% accuracy with respect to the Morrison's equation. Accordingly, using the Morrison's force amplitude at the mean sea level along with 90% of the total wave force obtained from the Morison's equation at the time when it is maximum yields the penetration depth; and hence, the dimensions of this linear distribution as follows (Figure 7).

$$0.9F_D^0 = \frac{f_0 z_0}{2} \tag{29}$$

$$0.9\left(\frac{\rho C_D D}{16k}(\omega H)^2\right) = 50\left(\frac{\omega H}{2}\right)^2 \frac{z_0}{2} \tag{30}$$

$$\frac{90}{16k}(\omega H)^2 = \frac{50}{8}(\omega H)^2 z_0 \tag{31}$$

$$\frac{0.9}{2\pi}\lambda = z_0 \tag{32}$$

$$z_0 = 0.15\lambda \tag{33}$$

$$\left.\begin{array}{l} \omega^2 = gk \rightarrow T^2 = 0.64\lambda \\ T^2 = 8.6H \end{array}\right\} \rightarrow \lambda = 13.5H \tag{34}$$

$$z_0 = 2H \tag{35}$$

As a result, the maximum estimated magnitude of the base shear force resulted from the net force acting at the one-third of the segment length from above and the total overturning moment of the wave corresponding to this linear distribution approximation about the base can be obtained as follows:

$$\begin{aligned} F^{Tri.} &\approx 50 v_0^2 H \\ F^{Tri.}_{Streched} &\approx 62.5 v_0^2 H \end{aligned} \tag{36}$$

$$\begin{aligned} M^{Tri.} &\approx 50 v_0^2 H \left(d - \frac{2H}{3}\right) \\ M^{Tri.}_{Stretched} &\approx 62.5 v_0^2 H \left(d - \frac{H}{3}\right) \end{aligned} \tag{37}$$

where v_0 is determined based on Eqs. (26) and (27).

In the second method, the exponential distribution of the wave force in depth is replaced by uniformly varying distributed (UVD) load segments along the projected length of the members normal to the direction of wave propagation. Two scenarios are considered in this approach: (1) one (UVD1) and (2) two (UVD2) uniform distributed load segments with 90% and 99% accuracy in comparison with the Morrison's equation, respectively. In these scenarios, it is assumed that the penetration depth for both upper and lower segments are the same as the one obtained from the previous method (i.e., $z_0 = 2H$). Accordingly, the magnitude of the upper force distribution at the mean sea level for both first and second scenario is determined based on 90% and the lower force distribution magnitude is carried out based on 9% of the total wave force obtained from the Morison's equation in order to give 99% accuracy from the resultant net forces for the second scenario (Figures 8 and 9).

$$0.9F_D^0 = 2f_0^{\text{Upper Seg.}} H = 20 f_0^{\text{Lower Seg.}} H \tag{38}$$

$$\begin{aligned} &0.9\left(\frac{\rho C_D D}{16k}(\omega H)^2\right) \\ &= 2f_0^{\text{Upper Seg.}} H = 20 f_0^{\text{Lower Seg.}} H \end{aligned} \tag{39}$$

$$\left.\begin{array}{l} T^2 = 8.6H \\ Deep\,water \rightarrow \omega^2 = gk \end{array}\right\} \rightarrow \left\{\begin{array}{l} f_0^{\text{Upper Seg.}} = 25 v_0^2 \\ f_0^{\text{Lower Seg.}} = 1.5 v_0^2 \end{array}\right. \tag{40}$$

Therefore, the maximum estimated magnitude of the base shear force resulted from the net forces acting at the mid-length of the segments and the total overturning moment of the wave corresponding to these uniform distribution approximations about the base can be expressed as follows:

$$\begin{aligned} F^{\text{Upper Seg.}} &\approx 50 v_0^2 H \\ F^{\text{Upper Seg.}}_{Stretched} &\approx 62.5 v_0^2 H \end{aligned} \tag{41}$$

$$F^{\text{Lower Seg.}} \approx F^{\text{Lower Seg.}}_{Stretched} \approx 3 v_0^2 H \tag{42}$$

$$\begin{aligned} M^{\text{Upper Seg.}} &\approx 50 v_0^2 H \left(d - H\right) \\ M^{\text{Upper Seg.}}_{Stretched} &\approx 62.5 v_0^2 H \left(d - 3H/4\right) \end{aligned} \tag{43}$$

$$M^{\text{Lower Seg.}} \approx M^{\text{Lower Seg.}}_{Stretched} \approx 3 v_0^2 H \left(d - 3H\right) \tag{44}$$

where v_0 is determined based on Eqs. (26) and (27).

It should be stated that the penetration depths in both models considering the effects of free surface fluctuations are the same. The numerical procedure of developed simple methods are presented in Figure 6.

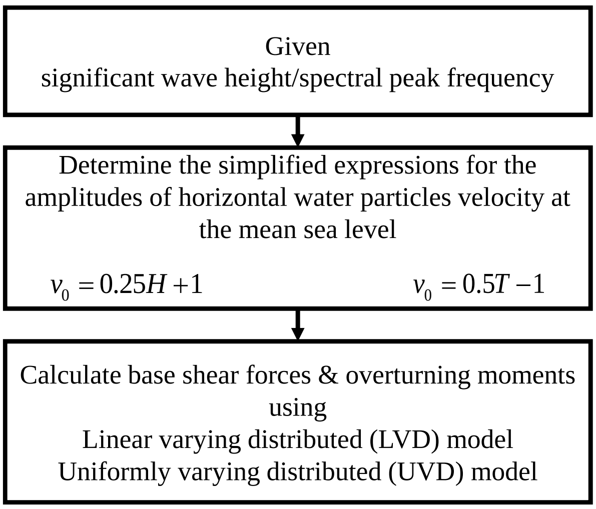
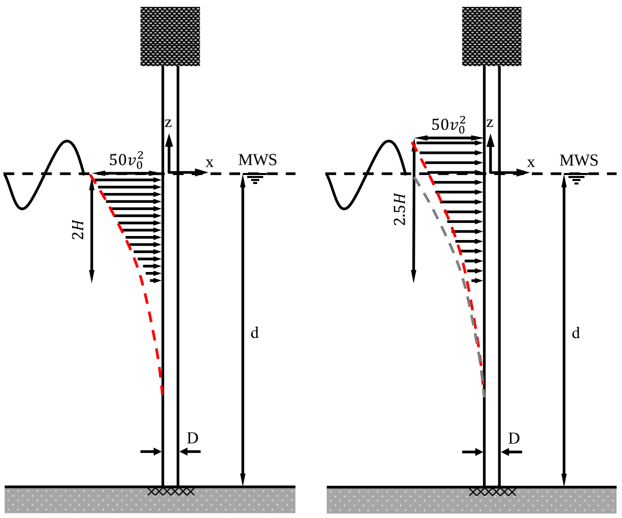
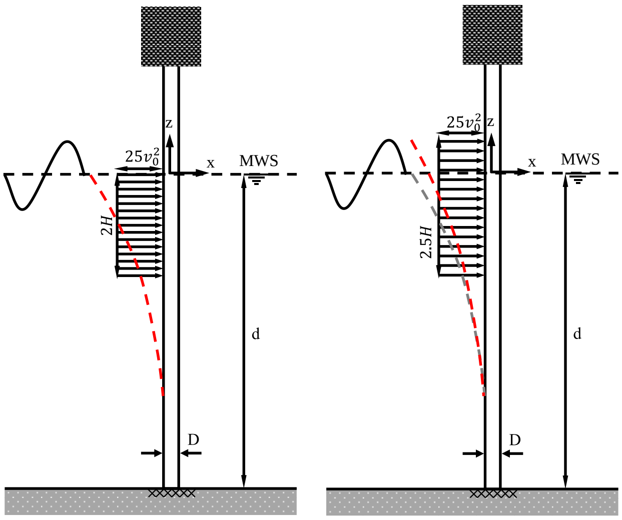


Figure 6. Schematic of developed simple models numerical procedure.



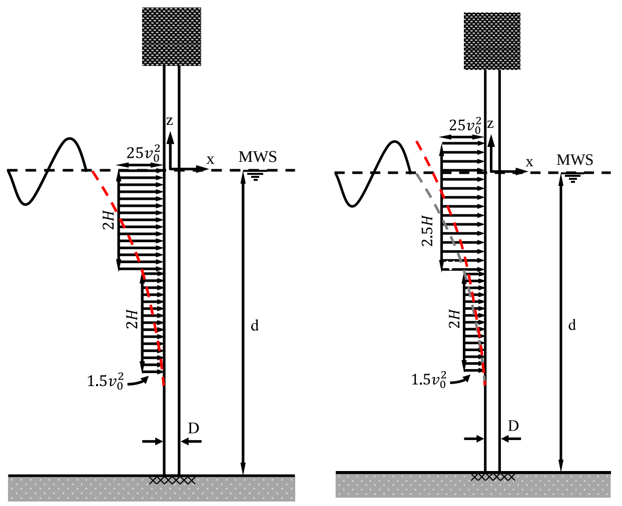
(a) (b)

Figure 7. The linear varying distributed (LVD) model (a) with and (b) without considering the effects of the instantaneous free water-wave surface.



(a) (b)

Figure 8. The single segment uniform varying distribution (UVD1) model (a) with and (b) without considering the effects of the instantaneous free water-wave surface.



(a) (b)

Figure 9. The double segments uniform varying distribution (UVD2) model (a) with and (b) without considering the effects of the instantaneous free water-wave surface.

6. Results and Discussion

6.1. Verification

In order to evaluate the accuracy of the proposed methods, the Demanding Parameters (DPs) (i.e., base shear and overturning moment) of a surface piercing vertical cantilever tabular pile foundation as a representative of the cylindrical shape structural components of the offshore platforms (e.g. monopile foundation, legs, braces, spokes and mooring lines) at 50 m depth are compared with the analytical nonlinear wave-induced forces and existing linearized models. For this purpose, a series of different sea states are considered to assess and verify the performance of these methods. The relevant parameters of the applied load cases are summarized in Table 1.

Table 1. Selected load cases.

Load case	H_s [m]	T_p [sec]
1	1	2.1
2	2	4.16
3	4	5.88
4	6	7.2
5	8	8.3
6	10	9.3
7	12	10.2
8	14	11
9	16	11.8

Figures 10 and 11 compare the computed base shear forces and the corresponding values of overturning moments using different methods without considering the effects of instantaneous free surface fluctuations. It is seen that the wave force distributions along the length of the member calculated based on the linear varying distributed (LVD) distribution approach and single segment uniformly varying distributed (UVD1) approach are in reasonable agreement with the values from conventional approach with little underestimation in all sea states. The comparison results from the double segments uniformly varying distributed (UVD2) approach and the analytical method also show that even though UVD2 method provides a good estimation of base shear forces in low and moderate sea states, the results of this method tend to deviate and overestimate the induced wave forces in the severe sea states mostly because of the overestimation in simplified water particle velocity expressions (see Figures 4 and 5). Regarding the overturning moment, UVD1 method result in a good estimation in all sea states, while the LVD and UVD2 methods yield nearly equal but overestimated especially in severe environmental conditions.

The computed Demanding Parameters using different methods considering the effects of instantaneous free surface fluctuations are also shown in Figures 12 and 13. The comparison results show that the overestimations in both approaches are in a good range of accuracy, while the calculation of overturning moments in LVD method seems to yield in higher but acceptable overestimation when the effects of free surface is considered. Moreover, it can be observed that both LVD and UVD approach give a better estimation than the linearized model using the Fourier series expansion. Therefore, it can be concluded that the proposed methods yield in reliable, but mostly conservative results which is acceptable in initial calculation design process.

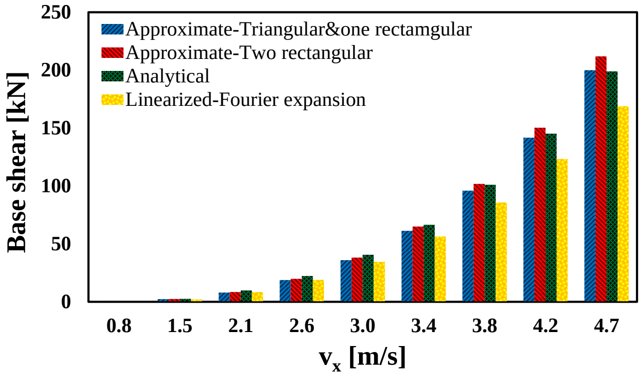


Figure 10. Computed base shear forces without considering the instantaneous free surface fluctuations for different load cases.

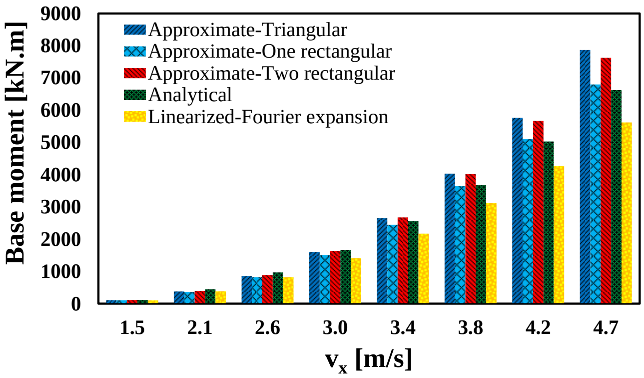


Figure 11. Computed overturning moments without considering the instantaneous free surface fluctuations for different load cases.

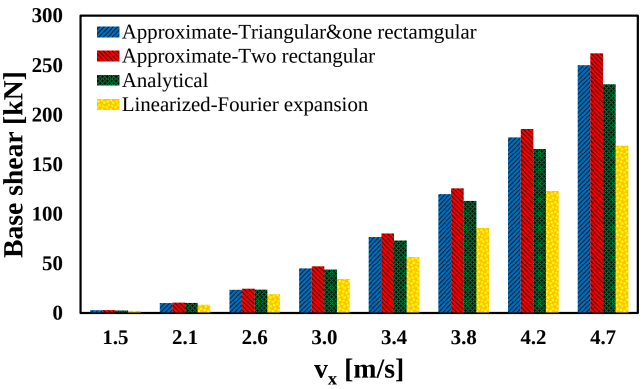


Figure 12. Computed base shear forces considering the instantaneous free surface fluctuations for different load cases.

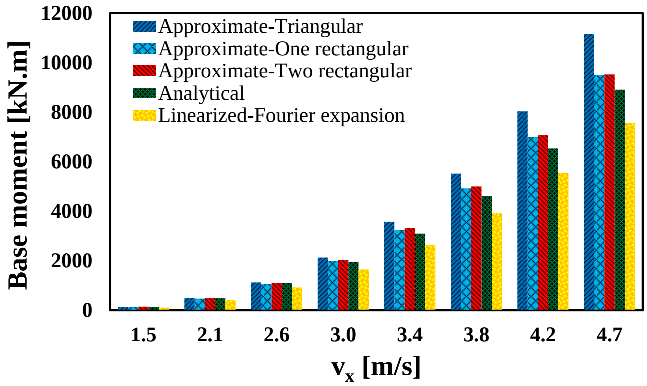


Figure 13. Computed overturning moments considering the instantaneous free surface fluctuations for different load cases.

6.2. Numerical case study

In addition to the previous section, a real scaled tested offshore jacket platform [36] is selected to evaluate the application of the simplified proposed methods in comparison with typical approach. The platform has four battered legs with diagonal brace members in vertical plans and is located in 4.88 m water depth. Structural elements perspective and properties of the platform is shown in Figure 14. The Airy wave theory with the wave height assumed to be equal to one-twentieth of the wave length was considered in all the perform tests presented in [36]. The experimental results were also carried out assuming inertia and drag coefficients to be equal to 2.0 and 1.0, respectively.

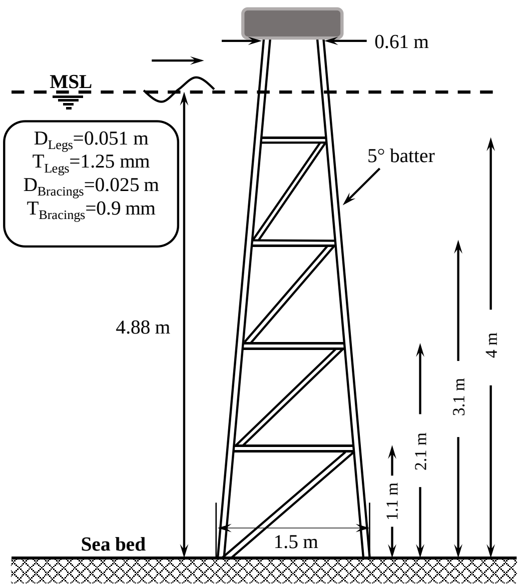


Figure 14. Schematic of the selected jacket platform and its structural properties.

To compute the total wave force (i.e., base shear) on the structure for both nonlinear and simplified proposed wave-induced forces, a three-dimensional (3D) numerical finite element model is developed using MatLab®. In this model, a linear interpolation between four calculation points along each element is considered to calculate the wave load on each member. Nevertheless, before comparing the results for the real platform, the accuracy of developed FEM model is investigated for the surface piercing vertical cantilever beam studied in the previous subsection. From Figure 15, it is can be observed that the developed FEM model results in reasonable values in both high and low frequencies compared to conventional and simplified approaches, which validate the developed numerical model.

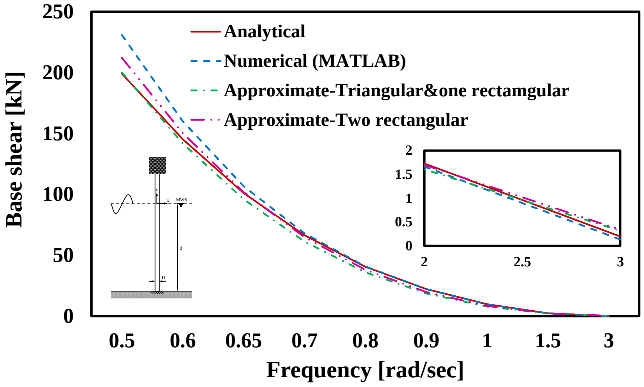


Figure 15. Verification of developed FEM model for base shear forces without considering the instantaneous free surface fluctuations for different excitation frequencies.

The developed finite element model is then used for the jacket platform (see Figure 16). The comparison with experimental data presented in Figure 17 shows a good match between the analytical, numerical, and simplified methods, which proves the accuracy of proposed methods in this study for a real platform. It should be stated that although the comparison result for the jacket platform does not validate the simplified methods developed in this study for lower frequency than 2 rad/sec due to unavailable experimental data, since the validation is examined for such low frequency range of a surface piercing vertical cantilever beam, it is assumed that these methods are applicable for low, moderate, and high excitation frequencies.

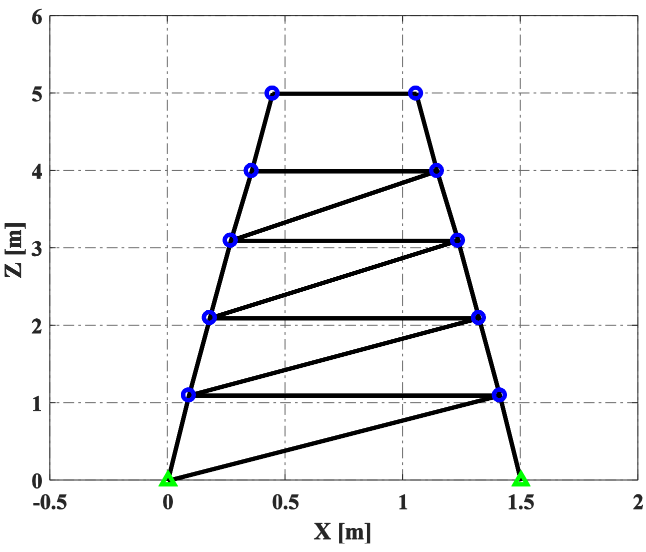


Figure 16. Two-dimensional view of the developed finite element model of the jacket platform.

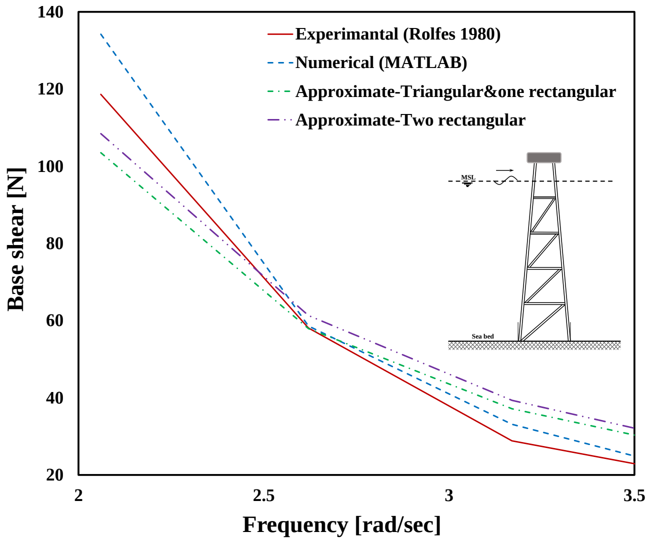


Figure 17. Comparison of computed base shear forces and experimental results of the small offshore test structure without considering the instantaneous free surface fluctuations.

7. Conclusions

This study presented details on driving new simplified approximation models incorporating the hydro dynamical wave forces calculation applied on framed members of offshore structures based on linear wave theory and the Morison's equation. In order to verify the accuracy of the proposed calculation methods, the static base shear force and overturning moment by the base line of a representative pile foundation/slender member are computed by the developed simplified expressions in different sea states and compared with the existing linearized model as well as the nonlinear Morison's equation. Moreover, the applicability of the proposed simplified methods is evaluated in comparison with experimental results of a real scaled tested offshore jacket platform. The results show that the achieved simplified models give good and realistic estimations of the maximum wave loads; and hence, corresponding maximum static base shear forces and overturning moments. Accordingly, utilizing such approximations not only make the hydro dynamical modeling and behaviour assessment of offshore support structures such as TLPs and semi-submersible platforms consist of large and small bodies more feasible and more implementable through mixed approaches (e.g., potential theory and Morison's equation) but also very viable for using in the first steps of design because of its smaller required computational time.

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