INTERNATIONAL JOURNAL OF
COASTAL. OFFSHORE & ENVIRONMENTAL ENGINEERING IJCOE Vol.9 /No.1 / winter 2024(1-24)

vailable online at: www. Ijcoe.org

A New Look at the Vertical Shear of the Geostrophic Current Part II: Thermal Current and Saline Current

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ARTICLE INFO

Article History:
Received: 14 Jan. 2024
Accepted: 09 Mar. 2024

Keywords:
Thermohaline circulation
Sensible heat transfer
Deep sea dynamics
Marine ecosystem
World climate balance

ABSTRACT

Many appropriate and necessary phenomena and mechanisms have essential roles for transfer and diffusion of arriving solar radiation. from tropical regions to high latitudes and low latitudes.
Atmospheric movements (winds) and oceanic movements (currents) are some parts of these mechanisms. Even different shear of them (movements) including vertical shear of them; have basic roles in the subject.
Our goal in this research is familiarizing with oceanic efforts for transfer of sensible heat via thermal current and saline current those are special cases of dense current and those are vertical shear of geostrophic current.
In connection with the subject; three versions of thermal current and three versions of saline current were introduced in Cartesian coordinates system and pressure coordinates system.
Furthermore; other subjects related to thermal current and saline current. have been discussed expanded upon the research. Also. study of thermal current and saline current can enlighten deep sea dynamics and help to better understanding climate of deep oceans.
Overall. these currents play a vital role in the ocean's ecosystem. climate and weather patterns. and human activities. They also have a major influence on the distribution of heat. carbon and other elements in the ocean. and they support marine life. Understanding these currents is important for managing resources and predicting future changes in the ocean's environment.

1. Introduction

In part I of "*A New Look at the Vertical Shear of the Geostrophic Current: Dense Current*" we referred to some theoretical aspects about vertical shear of the geostrophic current and number of basic ideas in connection with definition and deriving formulae to describe "*Dense Current*". [1]
Also. we pointed out majority of incoming solar radiation enter at tropical regions of the earth. Absorbed solar heat in tropical region. transfers or diffuses via some mechanisms. to high latitudes in northern hemisphere and low latitudes in southern hemisphere. The most important of these mechanisms are: winds. ocean currents. sensible heat. latent heat. monsoon phenomenon (as other type of latent heat transfer). tropical cyclones. meridional overturning circulation. Rossby waves and Antarctic circumpolar current.
These essential mechanisms; are agents of produce other sub-mechanisms or phenomena themselves too; those have a special role for heat transfer or heat diffusion directly or indirectly.
Especially; all atmospheric or oceanic movements. have a specific role or diverse roles for transferring heat from tropical regions to other parts of the earth. Even. variation of these movements. so that. assessed in this manner. For instance; variations of wind in vertical direction and variations of oceanic current with respect to depth; are inside of these struggles those arrived heat in the earth. doesn't remain in one region and transferred or diffused to another regions in order to earth will be usable for living human beings and other living beings.
Although variation of wind with respect to height investigated many times and. in this field. observational studies as well as researches have been

done since the beginning of the last century; variation of oceanic currents with respect to depth have been rarely investigated. For instance; recently. Zamanian introduced dense wind [2] and. thermal wind and moist wind [3] as results of vertical shear of geostrophic wind and showed how light air. dense air. heat or moisture advects in atmosphere to discover response time for returning of the; thermodynamic equilibrium.

Now. it is necessary to do researches and observational studies in this important field. i.e.. variations of oceanic currents with respect to depth. [1]

In the first place. variation of oceanic current is used for detection of advection of light or heavy seawater; and in the second stage. it helps to verify current observation and in the third place. detecting oceanic phenomenon type. because vertical shear of oceanic current will be happened in condition that ocean water is baroclinic and in this circumstance; we can track returning the oceanic thermodynamic equilibrium.

Now. some important points of part I are as follows:

1 – 1 – Theoretical study of vertical shear of the oceanic current is an urgent need for careful studies. to decipher many oceanic phenomena.

1 – 2 – We pointed out that various sensible heat flux or salinity flux can lead to produce baroclinic oceans and in turn; variation of geostrophic current in vertical direction. Furthermore. there is direct interaction between heat or salinity flux with current shear.

1 – 3 – After some description of geostrophic current; reason of variation of geostrophic current with respect to depth showed by logical argument and figure 1.

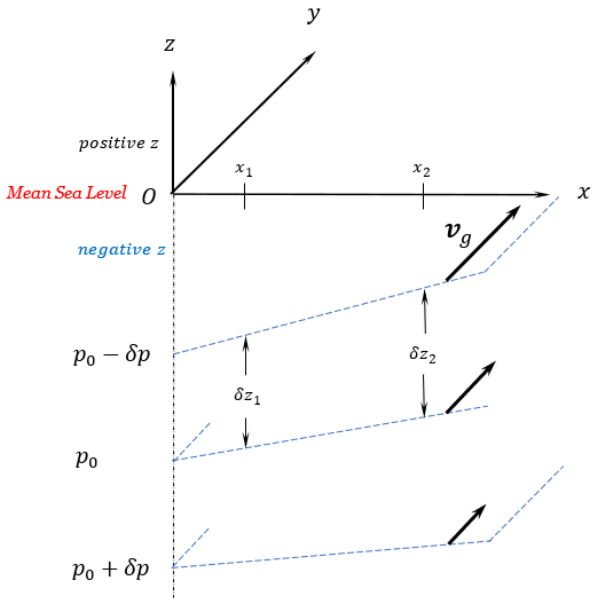

Figure 1. Relationship between vertical shear of the geostrophic current and horizontal thickness gradients. (Note that $0 < \delta p$) [1]

In subsequent time; we propounded pre-assumptions for dense current and focus to movement obtains power from horizontal gradient of density solely. In this manner; we excluded wind driven current. Ekman spiral. tidal current and river outflows. Likewise. we emphasized this current should start only by baroclinic ocean not other forces. Therefore; one can categorize dense current as deep current of ocean frequently.

Afterward; we defined "*Dense Current*" as**:**

"*Dense current is vectorial difference of geostrophic current vector at upper level and geostrophic current vector at lower level*" (of the oceanic layer). [see figure 2] providing no more forces to produce oceanic current same as wind. tidal force or river outflow exception horizontal gradient of density; that is:

$$\mathbb{v}_{\mathrm{D_c}} \equiv \mathbb{v}_{g(z_{2_a})} - \mathbb{v}_{g(z_{1_a})} \qquad \text{(1-Dc)}$$

where in equation (1-Dc) $\mathbb{v}_{\mathrm{D_c}}$ stands for dense current vector. $\mathbb{v}_{g(z_{2_a})}$ is geostrophic current at upper level (at less depth). and $\mathbb{v}_{g(z_{1_a})}$ is geostrophic current at lower level (at more depth) where z_{2_a} refers to level with less depth and z_{1_a} refers to level with more depth. i.e.. $z_{1_a} < z_{2_a}$. [see figure 2]

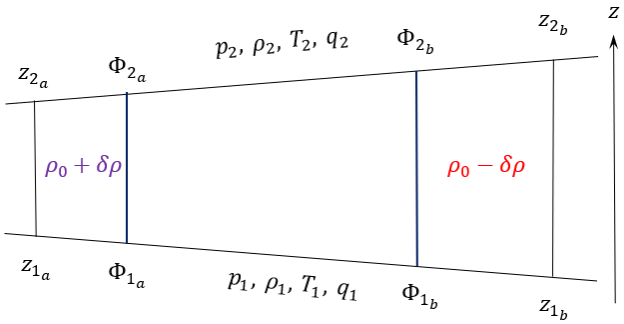

Figure 2. Typical cross section of atmospheric or oceanic layer. [1]

According to definition (1-Dc); eastward and northward components of dense current can be shown as following:

$$u_{\mathrm{D_c}} = u_{g(z_{2_a})} - u_{g(z_{1_a})} \qquad \text{(2-Dc-a)}$$

and

$$v_{\mathrm{D_c}} = v_{g(z_{2_a})} - v_{g(z_{1_a})} \qquad \text{(2-Dc-b)}$$

where in equation (2-Dc-a). $u_{\mathrm{D_c}}$ is eastward component of dense current. $u_{g(z_{2_a})}$ is eastward component of geostrophic current in less depth and $u_{g(z_{1_a})}$ is eastward component of geostrophic current in more depth.

In this manner; in equation (2-Dc-b). $v_{\mathrm{D_c}}$ is northward component of dense current. $v_{g(z_{2_a})}$ is northward component of geostrophic current in less depth and $v_{g(z_{1_a})}$ is northward component of geostrophic current in more depth.

Despite its name; dense current. while a vector. is not a true current. Instead. it is a geostrophic current shear. representing the change of geostrophic current with respect to depth (or height). causing some advections.

In the condition that wind blows on ocean surface; if environment is baroclinic. geostrophic current shear can obtain power from Ekman spiral as well as baroclinity[1] of domain. and variation of geostrophic current with respect to depth is resultant of both forces. In this condition; force of Ekman spiral – especially in less depth – has more power naturally; and furthermore. transfer of momentum from atmosphere to ocean frustrates geostrophic theory. However. in this work we assume wind doesn't blow on ocean surface and variations of geostrophic current with respect to depth. obtains power from baroclinity of domain solely. Particularly; this subject is true for neighborhood of 30 degrees latitudes both in northern hemisphere and southern hemisphere. when there is no atmospheric system and wind doesn't blow. Of course; observations should confirm this phenomenon.

Ekman spiral – dense current interaction is more precise discussion that is not the case for this work. and now we proceed to explain dense current only; on the conditions those are:

1 – 4 – Assuming that wind doesn't blow over ocean;

1 – 5 – Assuming that no more tidal force i.e.. in amphidromic region;

1 – 6 – Assuming that no more river's outflow;

1 – 7 – Assuming that no more other current in ocean's depths and

1 – 8 – Assuming the current is geostrophic and variation of geostrophic current with respect to depth enforces form baroclinic ocean only. (This case is true in the neighborhoods of latitudes -30 and +30. (But it should examine by observations.)

The above-mentioned assumptions are basics and true in the complete of this article.

1 – 9 – Furthermore. with using a number of formulae from a few text books same as text books of Pedlosky [4]. Holton and Hakim [5]. Apel [6]. Marshal and Plumb [7]. Gill [8]. Pedlosky [9]. Ghil and Simonnet [10] and Simmons [11]; we derived three versions of dense current's equations. vectors and components. [1]

1 – 10 – For the first version of dense current; dense current equation was:

$$\frac{\partial \mathbb{v}_g}{\partial z} = -\frac{g}{\rho f}(\mathbb{k} \times \nabla\rho) \qquad (3\text{-Dc-I})$$

We name equation (3-Dc-I) as the ***First version of dense current equation.*** Where in equation (3-Dc-I). $\mathbb{v}_g$ is geostrophic current. z is vertical axis of Cartesian coordinate system. g is acceleration due to gravity. ρ is density of seawater. f is Coriolis parameter. $\mathbb{k}$ stands for vertical unit vector in Cartesian coordinates system and ∇ is gradient operator in this coordinates system.

Equation (3-Dc-I) shows when geostrophic current can change in vertical direction. or when we have dense current that; we have horizontal gradient of density. In other word; in the case. ocean medium should be baroclinic.

Hereafter. we refer to z_1 as level with more depth of the oceanic layer and to z_2 as level with less depth of the oceanic layer. i.e.. $z_1 < z_2$.

Furthermore; we can derive ***the first version of dense current vector*** by integration of equation (3-Dc-I) from higher depth z_1 to lower depth z_2 in vertical direction as following: (by pay attention to figure 2)

$$\mathbb{v}_{D_{c_I}} = -\frac{g}{f}\int_{z_1}^{z_2}\frac{1}{\rho}(\mathbb{k} \times \nabla\rho)dz \qquad (4\text{-Dc-I})$$

In equation (4-Dc-I) or equation for the first version of dense current vector; $\mathbb{v}_{D_{c_I}}$ is the first version of dense current vector. z_1 is lower level of oceanic layer. z_2 is higher level of oceanic layer. ∇ is gradient operator and other parameters defined after equation (3-Dc-I). For equation (4-Dc-I) – that is the equation of the first version of dense current vector – we used definition (1-Dc).

Eastward and northward components of the first version of dense current were:

$$u_{D_{c_I}} = \frac{g}{f}\int_{z_1}^{z_2}\frac{1}{\rho}\frac{\partial\rho}{\partial y}dz \qquad (5\text{-Dc-I-a})$$

and:

$$v_{D_{c_I}} = -\frac{g}{f}\int_{z_1}^{z_2}\frac{1}{\rho}\frac{\partial\rho}{\partial x}dz \qquad (5\text{-Dc-I-b})$$

where in equation (5-Dc-I-a) $u_{D_{c_1}}$ is eastward component of the first version of dense current and in equation (5-Dc-I-b) $v_{D_{c_1}}$ is northward component of the first version of dense current. In addition. we used definitions (2-Dc-a) and (2-Dc-b) for deriving these formulae.

From the first version of dense current vector or its components; one can find out that:

1 – 10 – A: Whatever we go from pole to equator. dense current becomes stronger[2];

1 – 10 – B: If the oceanic layer has more thickness; then dense current becomes more powerful;

1 – 10 – C: Whatever oceanic layer is near to ocean surface. density would be lower and dense current becomes stronger as a result. and

1 – 10 – D: If the horizontal gradient of density would be greater. dense current becomes more powerful. because dense current is proportional to the horizontal gradient of density.

[1] Baroclinity is "*the existence of horizontal gradient of density.*" For oceanic medium; the horizontal gradient of density is related to the horizontal gradient of temperature. the horizontal gradient of salinity or the horizontal gradient of pressure; in general. all of them.

[2] Use of geostrophic current in tropical regions must be with careful deliberation because geostrophic current in these regions is magnified and especially on equator is meaningless.

In this manner. the first version of dense current vector. i.e.. equation (4-Dc-I) shows that: "*The first version of dense current flows parallel to Isopycnals. so that. low density is located at the right side of downstream.*" (In the northern hemisphere)
Furthermore; In this case. counterclockwise turning of geostrophic current with respect to depth (backing) is associated with lower density of seawater advection by geostrophic current in the layer.
In other words; backing of geostrophic current with respect to depth associated with advection of lower density of seawater – that may be from warmer or fresher seawater – in the oceanic layer.
Conversely; clockwise turning of geostrophic current with respect to depth (veering) implies advection of seawater with higher density by geostrophic current in the oceanic layer. In other words; veering of geostrophic current with respect to depth associated with advection of seawater with higher density – that may be from colder or saltier seawater – in the oceanic layer. [1]
1 – 11 – For the second version of dense current; dense current equation was:

$$\mathbb{v}_{\mathrm{D}_{\mathrm{c_{II}}}} = -\frac{1}{f}\int_{z_1}^{z_2}\frac{1}{\rho}(\mathbb{k}\times\nabla\rho)\,d\Phi \qquad \text{(4-Dc-II)}$$

In equation (4-Dc-II) that is: <u>*The second version of dense current vector*</u>: $\mathbb{v}_{\mathrm{D}_{\mathrm{c_{II}}}}$ is the second version of dense current vector. f is Coriolis parameter. z_1 is depth of lower level of oceanic layer. z_2 is depth of higher level of oceanic layer. ρ is density of seawater. $\mathbb{k}$ is vertical unit vector in Cartesian coordinates system. ∇ is gradient operator in Cartesian coordinates system and Φ is geopotential. Equation (4-Dc-II) is the second version of equation of dense current vector and we have used definition (1-Dc) for extracting it.
Eastward and northward components of the second version of dense current were:

$$u_{\mathrm{D}_{\mathrm{c_{II}}}} = \frac{1}{f}\int_{z_1}^{z_2}\frac{1}{\rho}\frac{\partial\rho}{\partial y}d\Phi \qquad \text{(5-Dc-II-a)}$$

and:

$$v_{\mathrm{D}_{\mathrm{c_{II}}}} = -\frac{1}{f}\int_{z_1}^{z_2}\frac{1}{\rho}\frac{\partial\rho}{\partial x}d\Phi \qquad \text{(5-Dc-II-b)}$$

where in equation (5-Dc-II-a). $u_{\mathrm{D}_{\mathrm{c_{II}}}}$ is eastward component of the second version of dense current and we have used definition (2-Dc-a) for deriving it. And in equation (5-Dc-II-b). $v_{\mathrm{D}_{\mathrm{c_{II}}}}$ is northward component of the second version of dense current and we have used definition (2-Dc-b) for extracting this equation.

All specifications. properties and figures of the second version of dense current are the same as the first version of dense current approximately. [1]
1 – 12 – For the third version of dense current; dense current vector was:

$$\mathbb{v}_{\mathrm{D}_{\mathrm{c_{III}}}} = f^{-1}\mathbb{k}\times\nabla_p(\Phi_2-\Phi_1) \qquad \text{(4-Dc-III)}$$

where. in equation (4-Dc-III). $\mathbb{v}_{\mathrm{D}_{\mathrm{c_{III}}}}$ is: <u>*The third version of dense current vector*</u>. f is Coriolis parameter. $\mathbb{k}$ is vertical unit vector in pressure coordinates system[3]. ∇_p is gradient operator in pressure coordinates system. Φ_2 is geopotential of higher level of the oceanic layer. Φ_1 is geopotential of lower level of the oceanic layer and $(\Phi_2-\Phi_1)$ is geopotential thickness of the oceanic layer.
Also; eastward component of the third version of dense current is:

$$u_{\mathrm{D}_{\mathrm{c_{III}}}} = -\frac{1}{f}\frac{\partial}{\partial y}(\Phi_2-\Phi_1) \qquad \text{(5-Dc-III-a)}$$

In equation (5-Dc-III-a). $u_{\mathrm{D}_{\mathrm{c_{III}}}}$ is eastward component of the third version of dense current and we used definition (2-Dc-a) for deriving this equation.
And northward component of the third version of dense current is:

$$v_{\mathrm{D}_{\mathrm{c_{III}}}} = \frac{1}{f}\frac{\partial}{\partial x}(\Phi_2-\Phi_1) \qquad \text{(5-Dc-III-b)}$$

In equation (5-Dc-III-b). $v_{\mathrm{D}_{\mathrm{c_{III}}}}$ is northward component of the third version of dense current and we used definition (2-Dc-b) for deriving this equation.
From the third version of dense current vector or its components; one can find out that:
1 – 12 – A: Whatever we go from pole to equator. the third version of dense current becomes stronger (with pay attention to footnote No. 2)
1 – 12 – B: If the oceanic layer has more geopotential thickness; then the third version of the dense current becomes more powerful. and
1 – 12 – C: If the horizontal gradient of geopotential thickness of the oceanic layer would be greater; then the third version of the dense current becomes more powerful. because this version of dense current is proportional to the horizontal gradient of geopotential thickness.
In this manner. the third version of dense current vector. i.e.. equation (4-Dc-III) shows that: "*The third version of dense current flows parallel to isopleths of geopotential thickness of the oceanic layer so that. high thickness of the layer is located at the right side of downstream.*" (In the northern hemisphere)

[3] In this work; whenever we refer to *"pressure coordinates system*" our purpose is "*Cartesian coordinates system with pressure as vertical coordinate"*. Note this coordinates system is "*Left-handed system*" and some text book referred to it as "*Isobaric coordinates system*"

In connection with the third version of dense current; counterclockwise turning of geostrophic current with respect to depth (backing) is associated with more thickness of oceanic layer advection by geostrophic current in the layer.
And; clockwise rotation of geostrophic current with respect to depth (veering) implies advection of less thickness of oceanic layer.
Therefore. it is possible to obtain a reasonable estimate of the horizontal advection of oceanic layer thickness and its vertical dependence at a given location solely from data on the vertical profile of the current given by current meter. Alternatively. the geostrophic current at any level of the oceanic layer can be estimated from the advection of less or more thickness of oceanic layer. provided that the geostrophic velocity is known at other level of that oceanic layer. [1]
Dense current is outstanding example of thermohaline circulation. Because. thermohaline circulation is the circulation of ocean water that is driven by differences in water density.
Now. following part I; in this part; we focus on special cases of dense current.

2. Special cases of Dense Current

The equation of state of seawater shows seawater density is depended on salinity. temperature and pressure. i.e.: [1]. [6]. [8]. [12] and [13]

$$\rho = \rho(s.T.p) \tag{6}$$

Equation of state for seawater is introduced in many texts same as [6]. [8] and [13] from origin [14] by following equation:

$$\rho = \rho(s.T.p) = \frac{\rho(s.T.0)}{1 - {}^{p}/_{K_T(s.T.p)}} \tag{7}$$

where in equation (7). ρ is density of seawater measuring by kgm^{-3}. s is salinity of seawater measuring by psu. T is temperature of seawater measuring by Celsius degrees ℃. p is pressure of seawater at defined level (depth) measuring by bar (10^5 Pa = $10^5 \mathrm{Nm}^{-2}$ = 1 bar). $K_T = K_T(s.T.p)$ is the mean bulk modulus which is inversely proportional to the compressibility of seawater and 0 in $\rho(s.T.0)$ refers to zero pressure at ocean surface. Each quantity on the right-hand side of (7). except pressure. is expressed as a polynomial series in s and T. expanded about values for zero salinity and a pressure of 1 bar.
The seawater density at the surface pressure ($p = 0$) is given by polynomial form:

$$\rho(s.T.0) = A + Bs + Cs^{3/2} + Ds^2 \tag{8}$$

And the mean bulk modulus is given by:

$$K_T(s.T.p) = E + Fs + Gs^{3/2} + \left(H + Is + Js^{3/2}\right)p + (M + Ns)p^2 \tag{9}$$

The coefficients $A.B.\ldots N$ in (8) and (9) are polynomial up to fifth degree in temperature. [14] The equation of state for seawater is very complicated and has accuracy about $0.009\ \mathrm{kgm}^{-3}$ over the entire oceanic pressure range. Some text introduces simple empirical expression for this equation. anyhow. [12]
The coefficients of polynomials are described as following:

$$A = 9.99842594 \times 10^2 + 6.793952 \times 10^{-2}T - 9.095290 \times 10^{-3}T^2 + 1.001685 \times 10^{-4}T^3 - 1.120083 \times 10^{-6}T^4 + 6.536332 \times 10^{-9}T^5 \tag{10}$$

$$B = 8.24493 \times 10^{-1} - 4.0899 \times 10^{-3}T + 7.6438 \times 10^{-5}T^2 - 8.2467 \times 10^{-7}T^3 + 5.3875 \times 10^{-9}T^4 \tag{11}$$

$$C = -5.72466 \times 10^{-3} + 1.0227 \times 10^{-4}T - 1.6546 \times 10^{-6}T^2 \tag{12}$$

$$D = 4.8314 \times 10^{-4} \tag{13}$$

$$E = 1.965221 \times 10^4 + 1.484206 \times 10^2T - 2.327105T^2 + 1.360477 \times 10^{-2}T^3 - 5.155288 \times 10^{-5}T^4 \tag{14}$$

$$F = 5.46746 \times 10^1 - 6.03459 \times 10^{-1}T + 1.09987 \times 10^{-2}T^2 - 6.1670 \times 10^{-5}T^3 \tag{15}$$

$$G = 7.944 \times 10^{-2} + 1.6483 \times 10^{-2}T - 5.3009 \times 10^{-4}T^2 \tag{16}$$

$$H = 3.239908 + 1.43713 \times 10^{-3}T + 1.16092 \times 10^{-4}T^2 - 5.77905 \times 10^{-7}T^3 \tag{17}$$

$$I = 2.2838 \times 10^{-3} - 1.0981 \times 10^{-5}T - 1.6078 \times 10^{-6}T^2 \tag{18}$$

$$J = 1.91075 \times 10^{-4} \tag{19}$$

$$M = 8.50935 \times 10^{-5} - 6.12293 \times 10^{-6}T + 5.2787 \times 10^{-8}T^2 \tag{20}$$

and:

$$N = -9.9348 \times 10^{-7} + 2.0816 \times 10^{-8}T + 9.1697 \times 10^{-10}T^2 \tag{21}$$

In part I of "*A New Look at the Vertical Shear of Geostrophic Current: Dense Current*" we referred to eastward and northward components of the first version of dense current as following [1]:

$$u_{\mathrm{D_{c_I}}} = \frac{g}{f}\int_{z_1}^{z_2} \frac{1}{\rho}\frac{\partial \rho}{\partial y}dz \tag{5-Dc-I-a}$$

and:

$$v_{\mathrm{D}_{\mathrm{c_I}}} = -\frac{g}{f}\int_{\mathrm{z}_1}^{\mathrm{z}_2}\frac{1}{\rho}\frac{\partial\rho}{\partial x}dz \qquad \text{(5-Dc-I-b)}$$

At the present time; we substitute equivalent of density ρ from equations (8) and (9) by substituting coefficients introduced in equations (10) – (21) into equation (5-Dc-I-a) and get:

$$u_{\mathrm{D}_{\mathrm{c_I}}} = \frac{g}{f}\int_{\mathrm{z}_1}^{\mathrm{z}_2}\frac{1}{\rho}\frac{\partial\rho}{\partial y}dz = \frac{g}{f}\int_{\mathrm{z}_1}^{\mathrm{z}_2}\frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}}\frac{\partial}{\partial y}\left[\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}\right]dz \qquad \text{(22-a)}$$

or:

$$u_{\mathrm{D}_{\mathrm{c_I}}} = \frac{g}{f}\int_{z_1}^{z_2}\frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}}\times\frac{\frac{\partial}{\partial y}\rho(s.T.0)\left[1-{}^{p}/_{K_T(s.T.p)}\right]-\rho(s.T.0)\frac{\partial}{\partial y}\left[1-{}^{p}/_{K_T(s.T.p)}\right]}{\left[1-{}^{p}/_{K_T(s.T.p)}\right]^2}dz \qquad \text{(23-a)}$$

But:

$$\frac{\partial}{\partial y}\left[1-{}^{p}/_{K_T(s.T.p)}\right] = -\frac{\partial}{\partial y}\left[{}^{p}/_{K_T(s.T.p)}\right] = \frac{p\frac{\partial}{\partial y}K_T(s.T.p)-\frac{\partial p}{\partial y}K_T(s.T.p)}{[K_T(s.T.p)]^2} \qquad \text{(24-a)}$$

Accordingly:

$$u_{\mathrm{D}_{\mathrm{c_I}}} = \frac{g}{f}\int_{z_1}^{z_2}\frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}}\times\frac{\frac{\partial}{\partial y}\rho(s.T.0)\left[1-{}^{p}/_{K_T(s.T.p)}\right]-\rho(s.T.0)(24-\mathrm{a})}{\left[1-{}^{p}/_{K_T(s.T.p)}\right]^2}dz \qquad \text{(25-a)}$$

where in equation (25-a). (24-a) refers to equation with the same name.
Now we will proceed every term. one by one.
By aid of equation (8). we have:

$$\frac{\partial}{\partial y}\rho(s.T.0) = \frac{\partial}{\partial y}\left(A+Bs+Cs^{3}/_{2}+Ds^2\right) = \frac{\partial}{\partial y}A+\frac{\partial}{\partial y}(Bs)+\frac{\partial}{\partial y}\left(Cs^{3}/_{2}\right)+\frac{\partial}{\partial y}(Ds^2) \qquad \text{(26-a)}$$

With pay attention to equation (10):

$$\frac{\partial}{\partial y}A = \frac{\partial}{\partial y}(9.99842594\times10^{2}+6.793952\times10^{-2}T-9.095290\times10^{-3}T^2+1.001685\times10^{-4}T^3-1.120083\times10^{-6}T^4+6.536332\times10^{-9}T^5) \qquad (27)$$

Therefore:

$$\frac{\partial}{\partial y}A = 6.793952\times10^{-2}\frac{\partial T}{\partial y}-1.819058\times10^{-2}T\frac{\partial T}{\partial y}+3.005055\times10^{-4}T^2\frac{\partial T}{\partial y}-4.480332\times10^{-4}T^3\frac{\partial T}{\partial y}+3.268166\times10^{-8}T^4\frac{\partial T}{\partial y} \qquad \text{(28-a)}$$

And with pay attention to equation (11):

$$\frac{\partial}{\partial y}(Bs) = \frac{\partial B}{\partial y}s+B\frac{\partial s}{\partial y} = \frac{\partial}{\partial y}(8.24493\times10^{-1}-4.0899\times10^{-3}T+7.6438\times10^{-5}T^2-8.2467\times10^{-7}T^3+5.3875\times10^{-9}T^4)s+(8.24493\times10^{-1}-4.0899\times10^{-3}T+7.6438\times10^{-5}T^2-8.2467\times10^{-7}T^3+5.3875\times10^{-9}T^4)\frac{\partial s}{\partial y} \qquad (29)$$

Consequently:

$$\frac{\partial}{\partial y}(Bs) = \left(-4.0899\times10^{-3}\frac{\partial T}{\partial y}+1.52876\times10^{-4}T\frac{\partial T}{\partial y}-2.47401\times10^{-6}T^2\frac{\partial T}{\partial y}+2.155\times10^{-8}T^3\frac{\partial T}{\partial y}\right)s+(8.24493\times10^{-1}-4.0899\times10^{-3}T+7.6438\times10^{-5}T^2-8.2467\times10^{-7}T^3+5.3875\times10^{-9}T^4)\frac{\partial s}{\partial y} \qquad \text{(30-a)}$$

In addition. by pay attention to equation (12):

$$\frac{\partial}{\partial y}\left(Cs^{3}/_{2}\right) = \frac{\partial C}{\partial y}\left(s^{3}/_{2}\right)+C\frac{\partial}{\partial y}\left(s^{3}/_{2}\right) = \frac{\partial}{\partial y}(-5.72466\times10^{-3}+1.0227\times10^{-4}T-1.6546\times10^{-6}T^2)\left(s^{3}/_{2}\right)+(-5.72466\times10^{-3}+1.0227\times10^{-4}T-1.6546\times10^{-6}T^2)\frac{\partial}{\partial y}\left(s^{3}/_{2}\right) \qquad (31)$$

Accordingly:

$$\frac{\partial}{\partial y}\left(Cs^{3}/_{2}\right) = \left(1.0227\times10^{-4}\frac{\partial T}{\partial y}-3.3092\times10^{-6}T\frac{\partial T}{\partial y}\right)\left(s^{3}/_{2}\right)+(-5.72466\times10^{-3}+1.0227\times10^{-4}T-1.6546\times10^{-6}T^2)\left(\frac{3}{2}\,s^{1}/_{2}\frac{\partial s}{\partial y}\right) \qquad \text{(32-a)}$$

And with pay attention to equation (13):

$$\frac{\partial}{\partial y}(Ds^2) = \frac{\partial}{\partial y}[(4.8314\times10^{-4})s^2] = 9.6628\times10^{-4}s\frac{\partial s}{\partial y} \qquad \text{(33-a)}$$

Therefore. derivative of seawater density at surface of ocean with respect to y. is organized by addition of equations (28-a). (30-a). (32-a) and (33-a). i.e.:

$$\frac{\partial}{\partial y}\rho(s.T.0) = (28-\mathrm{a})+(30-\mathrm{a})+(32-\mathrm{a})+(33-\mathrm{a}) \qquad \text{(34-a)}$$

Furthermore. with aid of equation (9) we have:

$$\frac{\partial}{\partial y}[K_T(s.T.p)] = \frac{\partial}{\partial y}(E) + \frac{\partial}{\partial y}(Fs) + \frac{\partial}{\partial y}\left(Gs^{3}/_{2}\right) + \frac{\partial}{\partial y}(Hp) + \frac{\partial}{\partial y}(Isp) + \frac{\partial}{\partial y}\left(Js^{3}/_{2}p\right) + \frac{\partial}{\partial y}(Mp^2) + \frac{\partial}{\partial y}(Nsp^2) \quad (35\text{-a})$$

And with aid of equation (l4). we have:

$$\frac{\partial}{\partial y}(E) = \frac{\partial}{\partial y}(1.965221 \times 10^4 + 1.484206 \times 10^2 T - 2.327105T^2 + 1.360477 \times 10^{-2}T^3 - 5.155288 \times 10^{-5}T^4) = 1.484206 \times 10^2 \frac{\partial T}{\partial y} - 4.65421T\frac{\partial T}{\partial y} + 4.081431 \times 10^{-2}T^2\frac{\partial T}{\partial y} - 2.0621152 \times 10^{-4}T^3\frac{\partial T}{\partial y} \quad (36\text{-a})$$

With help of equation (15). we get:

$$\frac{\partial}{\partial y}(Fs) = \frac{\partial F}{\partial y}s + F\frac{\partial s}{\partial y} = \frac{\partial}{\partial y}(5.46746 \times 10^1 - 6.03459 \times 10^{-1}T + 1.09987 \times 10^{-2}T^2 - 6.1670 \times 10^{-5}T^3)s + (5.46746 \times 10^1 - 6.03459 \times 10^{-1}T + 1.09987 \times 10^{-2}T^2 - 6.1670 \times 10^{-5}T^3)\frac{\partial s}{\partial y} \quad (37)$$

or:

$$\frac{\partial}{\partial y}(Fs) = \left(-6.03459 \times 10^{-1}\frac{\partial T}{\partial y} + 2.19974 \times 10^{-2}T\frac{\partial T}{\partial y} - 1.8501 \times 10^{-4}T^2\frac{\partial T}{\partial y}\right)s + (5.46746 \times 10^1 - 6.03459 \times 10^{-1}T + 1.09987 \times 10^{-2}T^2 - 6.1670 \times 10^{-5}T^3)\frac{\partial s}{\partial y} \quad (38\text{-a})$$

And with aid of equation (16). we have:

$$\frac{\partial}{\partial y}\left(Gs^{3}/_{2}\right) = \frac{\partial G}{\partial y}s^{3}/_{2} + G\frac{\partial}{\partial y}\left(s^{3}/_{2}\right) = \frac{\partial}{\partial y}(7.944 \times 10^{-2} + 1.6483 \times 10^{-2}T - 5.3009 \times 10^{-4}T^2)s^{3}/_{2} + (7.944 \times 10^{-2} + 1.6483 \times 10^{-2}T - 5.3009 \times 10^{-4}T^2)\frac{\partial}{\partial y}\left(s^{3}/_{2}\right) \quad (39)$$

Therefore:

$$\frac{\partial}{\partial y}\left(Gs^{3}/_{2}\right) = \left(1.6483 \times 10^{-2}\frac{\partial T}{\partial y} - 1.06018 \times 10^{-3}T\frac{\partial T}{\partial y}\right)s^{3}/_{2} + (7.944 \times 10^{-2} + 1.6483 \times 10^{-2}T - 5.3009 \times 10^{-4}T^2)\frac{3}{2}s^{1}/_{2}\frac{\partial s}{\partial y} \quad (40\text{-a})$$

Likewise. with help of equation (17). we get:

$$\frac{\partial}{\partial y}(Hp) = \frac{\partial H}{\partial y}p + H\frac{\partial p}{\partial y} = \frac{\partial}{\partial y}(3.239908 + 1.43713 \times 10^{-3}T + 1.16092 \times 10^{-4}T^2 - 5.77905 \times 10^{-7}T^3)p + (3.239908 + 1.43713 \times 10^{-3}T + 1.16092 \times 10^{-4}T^2 - 5.77905 \times 10^{-7}T^3)\frac{\partial p}{\partial y} \quad (41)$$

Consequently:

$$\frac{\partial}{\partial y}(Hp) = \left(1.43713 \times 10^{-3}\frac{\partial T}{\partial y} + 2.32184 \times 10^{-4}T\frac{\partial T}{\partial y} - 1.733715 \times 10^{-6}T^2\frac{\partial T}{\partial y}\right)p + (3.239908 + 1.43713 \times 10^{-3}T + 1.16092 \times 10^{-4}T^2 - 5.77905 \times 10^{-7}T^3)\frac{\partial p}{\partial y} \quad (42\text{-a})$$

And with assistance of equation (18):

$$\frac{\partial}{\partial y}(Isp) = \frac{\partial I}{\partial y}sp + I\frac{\partial s}{\partial y}p + Is\frac{\partial p}{\partial y} = \frac{\partial}{\partial y}(2.2838 \times 10^{-3} - 1.0981 \times 10^{-5}T - 1.6078 \times 10^{-6}T^2)sp + (2.2838 \times 10^{-3} - 1.0981 \times 10^{-5}T - 1.6078 \times 10^{-6}T^2)\frac{\partial s}{\partial y}p + (2.2838 \times 10^{-3} - 1.0981 \times 10^{-5}T - 1.6078 \times 10^{-6}T^2)s\frac{\partial p}{\partial y} \quad (43)$$

Accordingly:

$$\frac{\partial}{\partial y}(Isp) = \left(-1.0981 \times 10^{-5}\frac{\partial T}{\partial y} - 3.2156 \times 10^{-6}T\frac{\partial T}{\partial y}\right)sp + (2.2838 \times 10^{-3} - 1.0981 \times 10^{-5}T - 1.6078 \times 10^{-6}T^2)\frac{\partial s}{\partial y}p + (2.2838 \times 10^{-3} - 1.0981 \times 10^{-5}T - 1.6078 \times 10^{-6}T^2)s\frac{\partial p}{\partial y} \quad (44\text{-a})$$

And with aid of equation (19):

$$\frac{\partial}{\partial y}\left(Js^{3}/_{2}p\right) = \frac{\partial J}{\partial y}s^{3}/_{2}p + J\frac{\partial\left(s^{3}/_{2}\right)}{\partial y}p + Js^{3}/_{2}\frac{\partial p}{\partial y} = 2.866125 \times 10^{-4}s^{1}/_{2}\frac{\partial s}{\partial y}p + (1.91075 \times 10^{-4})s^{3}/_{2}\frac{\partial p}{\partial y} \quad (45\text{-a})$$

By help of equation (20):

$$\frac{\partial}{\partial y}(Mp^2) = \frac{\partial M}{\partial y}p^2 + M\frac{\partial}{\partial y}(p^2) = \frac{\partial}{\partial y}(8.50935 \times 10^{-5} - 6.12293 \times 10^{-6}T + 5.2787 \times 10^{-8}T^2)p^2 + (8.50935 \times 10^{-5} - 6.12293 \times 10^{-6}T + 5.2787 \times 10^{-8}T^2)\frac{\partial(p^2)}{\partial y} \quad (46)$$

Therefore:

$$\frac{\partial}{\partial y}(Mp^2) = \left(-6.12293 \times 10^{-6}\frac{\partial T}{\partial y} + 1.05574 \times 10^{-7}T\frac{\partial T}{\partial y}\right)p^2 + 2.0 \times (8.50935 \times 10^{-5} - 6.12293 \times 10^{-6}T + 5.2787 \times 10^{-8}T^2)p\frac{\partial p}{\partial y} \quad (47\text{-a})$$

Also. with help of equation (21):

$$\frac{\partial}{\partial y}(Nsp^2) = \frac{\partial N}{\partial y}sp^2 + N\frac{\partial s}{\partial y}p^2 + Ns\frac{\partial}{\partial y}(p^2) = \frac{\partial}{\partial y}(-9.9348 \times 10^{-7} + 2.0816 \times 10^{-8}T + 9.1697 \times 10^{-10}T^2)sp^2 + (-9.9348 \times 10^{-7} + 2.0816 \times 10^{-8}T + 9.1697 \times 10^{-10}T^2)\frac{\partial s}{\partial y}p^2 + 2.0 \times$$

$$(-9.9348 \times 10^{-7} + 2.0816 \times 10^{-8}T + 9.1697 \times 10^{-10}T^2)sp\frac{\partial p}{\partial y} \quad (48)$$

Consequently:

$$\frac{\partial}{\partial y}(Nsp^2) = \left(2.0816 \times 10^{-8}\frac{\partial T}{\partial y} + 1.83394 \times 10^{-9}T\frac{\partial T}{\partial y}\right)sp^2 + (-9.9348 \times 10^{-7} + 2.0816 \times 10^{-8}T + 9.1697 \times 10^{-10}T^2)\frac{\partial s}{\partial y}p^2 + 2.0 \times (-9.9348 \times 10^{-7} + 2.0816 \times 10^{-8}T + 9.1697 \times 10^{-10}T^2)sp\frac{\partial p}{\partial y} \quad (49\text{-a})$$

Therefore; according to equations (35-a). (36-a). (38-a). (40-a). (42-a). (44-a). (45-a). (47-a) and (49-a):

$$\frac{\partial}{\partial y}[K_T(s.T.p)] = \text{(36-a) + (38-a) + (40-a) + (42-a) + (44-a) + (45-a) + (47-a) + (49-a).} \quad (50\text{-a})$$

Furthermore. considering equation (24-a). we have:

$$\frac{\partial}{\partial y}\left[1 - {}^{p}/_{K_T(s.T.p)}\right] = \frac{p\frac{\partial}{\partial y}K_T(s.T.p) - \frac{\partial p}{\partial y}K_T(s.T.p)}{[K_T(s.T.p)]^2} = \frac{p(50\text{-a}) - \frac{\partial p}{\partial y}K_T(s.T.p)}{[K_T(s.T.p)]^2} \quad (51\text{-a})$$

Substituting (34-a) and (51-a) in (25-a) yields:

$$u_{D_{c_I}} = \frac{g}{f}\int_{z_1}^{z_2} \frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}} \times \frac{(34\text{-a})\left[1-{}^{p}/_{K_T(s.T.p)}\right] - \rho(s.T.0)(51\text{-a})}{\left[1-{}^{p}/_{K_T(s.T.p)}\right]^2}dz \quad (52\text{-a})$$

where in equation (52-a). $u_{D_{c_I}}$ is eastward component of the first version of dense current. g is acceleration due to gravity. f is Coriolis parameter. z_1 is lower level of the oceanic layer. z_2 is higher level of the oceanic layer. $\rho(s.T.0)$ is density of surface ocean defined in (8). p is pressure of seawater at any point considered. $K_T(s.T.p)$ is mean bulk modulus defined in (9). (34-a) is equation with the same number and (51-a) is also equation with the same number and z stands for vertical axis of Cartesian coordinates system.

Now; we substitute equivalent of density ρ from equations (8) and (9) by substituting coefficients introduced in equations (10) – (21) into equation (5-Dc-I-b) and get:

$$v_{D_{c_I}} = -\frac{g}{f}\int_{z_1}^{z_2}\frac{1}{\rho}\frac{\partial \rho}{\partial x}dz = -\frac{g}{f}\int_{z_1}^{z_2}\frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}}\frac{\partial}{\partial x}\left[\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}\right]dz \quad (22\text{-b})$$

or:

$$v_{D_{c_I}} = -\frac{g}{f}\int_{z_1}^{z_2}\frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}} \times \frac{\frac{\partial}{\partial x}\rho(s.T.0)\left[1-{}^{p}/_{K_T(s.T.p)}\right] - \rho(s.T.0)\frac{\partial}{\partial x}\left[1-{}^{p}/_{K_T(s.T.p)}\right]}{\left[1-{}^{p}/_{K_T(s.T.p)}\right]^2}dz \quad (23\text{-b})$$

But:

$$\frac{\partial}{\partial x}\left[1 - {}^{p}/_{K_T(s.T.p)}\right] = -\frac{\partial}{\partial x}\left[{}^{p}/_{K_T(s.T.p)}\right] = \frac{p\frac{\partial}{\partial x}K_T(s.T.p) - \frac{\partial p}{\partial x}K_T(s.T.p)}{[K_T(s.T.p)]^2} \quad (24\text{-b})$$

Accordingly:

$$v_{D_{c_I}} = -\frac{g}{f}\int_{z_1}^{z_2}\frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}} \times \frac{\frac{\partial}{\partial x}\rho(s.T.0)\left[1-{}^{p}/_{K_T(s.T.p)}\right] - \rho(s.T.0)(24\text{-b})}{\left[1-{}^{p}/_{K_T(s.T.p)}\right]^2}dz \quad (25\text{-b})$$

where in equation (25-b). (24-b) refers to equation with the same name.

Now. we will proceed every term. one by one.

By aid of equation (8). we have:

$$\frac{\partial}{\partial x}\rho(s.T.0) = \frac{\partial}{\partial x}\left(A + Bs + Cs^{3}/_{2} + Ds^2\right) = \frac{\partial}{\partial x}A + \frac{\partial}{\partial x}(Bs) + \frac{\partial}{\partial x}\left(Cs^{3}/_{2}\right) + \frac{\partial}{\partial x}(Ds^2) \quad (26\text{-b})$$

With similar manner for eastward component of the first version of dense current. we get:

$$\frac{\partial}{\partial x}A = 6.793952 \times 10^{-2}\frac{\partial T}{\partial x} - 1.819058 \times 10^{-2}T\frac{\partial T}{\partial x} + 3.005055 \times 10^{-4}T^2\frac{\partial T}{\partial x} - 4.480332 \times 10^{-4}T^3\frac{\partial T}{\partial x} + 3.268166 \times 10^{-8}T^4\frac{\partial T}{\partial x} \quad (28\text{-b})$$

And:

$$\frac{\partial}{\partial x}(Bs) = \left(-4.0899 \times 10^{-3}\frac{\partial T}{\partial x} + 1.52876 \times 10^{-4}T\frac{\partial T}{\partial x} - 2.47401 \times 10^{-6}T^2\frac{\partial T}{\partial x} + 2.155 \times 10^{-8}T^3\frac{\partial T}{\partial x}\right)s + (8.24493 \times 10^{-1} - 4.0899 \times 10^{-3}T + 7.6438 \times 10^{-5}T^2 - 8.2467 \times 10^{-7}T^3 + 5.3875 \times 10^{-9}T^4)\frac{\partial s}{\partial x} \quad (30\text{-b})$$

Also:

$$\frac{\partial}{\partial x}\left(Cs^{3}/_{2}\right) = \left(1.0227 \times 10^{-4}\frac{\partial T}{\partial x} - 3.3092 \times 10^{-6}T\frac{\partial T}{\partial x}\right)\left(s^{3}/_{2}\right) + (-5.72466 \times 10^{-3} + 1.0227 \times 10^{-4}T - 1.6546 \times 10^{-6}T^2)\left(\frac{3}{2}\,s^{1}/_{2}\frac{\partial s}{\partial x}\right) \quad (32\text{-b})$$

And:

$$\frac{\partial}{\partial x}(D_S^2) = \frac{\partial}{\partial x}[(4.8314 \times 10^{-4})s^2] = 9.6628 \times 10^{-4} s \frac{\partial s}{\partial x} \quad (33\text{-b})$$

Therefore. derivative of seawater density at surface of ocean with respect to x. is organized by addition of equations (28-b). (30-b). (32-b) and (33-b). i.e.:

$$\frac{\partial}{\partial x}\rho(s.T.0) = (28\text{-b}) + (30\text{-b}) + (32\text{-b}) + (33\text{-b}) \quad (34\text{-b})$$

Furthermore. with aid of equation (9) we have:

$$\frac{\partial}{\partial x}[K_T(s.T.p)] = \frac{\partial}{\partial x}(E) + \frac{\partial}{\partial x}(Fs) + \frac{\partial}{\partial x}\left(Gs^{3/2}\right) + \frac{\partial}{\partial x}(Hp) + \frac{\partial}{\partial x}(Isp) + \frac{\partial}{\partial x}\left(Js^{3/2}p\right) + \frac{\partial}{\partial x}(Mp^2) + \frac{\partial}{\partial x}(Nsp^2) \quad (35\text{-b})$$

And with help of equation (l4). we have:

$$\frac{\partial}{\partial x}(E) = \frac{\partial}{\partial x}(1.965221 \times 10^4 + 1.484206 \times 10^2 T - 2.327105T^2 + 1.360477 \times 10^{-2}T^3 - 5.155288 \times 10^{-5}T^4) = 1.484206 \times 10^2 \frac{\partial T}{\partial x} - 4.65421T\frac{\partial T}{\partial x} + 4.081431 \times 10^{-2}T^2\frac{\partial T}{\partial x} - 2.0621152 \times 10^{-4}T^3\frac{\partial T}{\partial x} \quad (36\text{-b})$$

And:

$$\frac{\partial}{\partial x}(Fs) = \left(-6.03459 \times 10^{-1}\frac{\partial T}{\partial x} + 2.19974 \times 10^{-2}T\frac{\partial T}{\partial x} - 1.8501 \times 10^{-4}T^2\frac{\partial T}{\partial x}\right)s + (5.46746 \times 10^1 - 6.03459 \times 10^{-1}T + 1.09987 \times 10^{-2}T^2 - 6.1670 \times 10^{-5}T^3)\frac{\partial s}{\partial x} \quad (38\text{-b})$$

In addition:

$$\frac{\partial}{\partial x}\left(Gs^{3/2}\right) = \left(1.6483 \times 10^{-2}\frac{\partial T}{\partial x} - 1.06018 \times 10^{-3}T\frac{\partial T}{\partial x}\right)s^{3/2} + (7.944 \times 10^{-2} + 1.6483 \times 10^{-2}T - 5.3009 \times 10^{-4}T^2)\frac{3}{2}s^{1/2}\frac{\partial s}{\partial x} \quad (40\text{-b})$$

And:

$$\frac{\partial}{\partial x}(Hp) = \left(1.43713 \times 10^{-3}\frac{\partial T}{\partial x} + 2.32184 \times 10^{-4}T\frac{\partial T}{\partial x} - 1.733715 \times 10^{-6}T^2\frac{\partial T}{\partial x}\right)p + (3.239908 + 1.43713 \times 10^{-3}T + 1.16092 \times 10^{-4}T^2 - 5.77905 \times 10^{-7}T^3)\frac{\partial p}{\partial x} \quad (42\text{-b})$$

Also:

$$\frac{\partial}{\partial x}(Isp) = \left(-1.0981 \times 10^{-5}\frac{\partial T}{\partial x} - 3.2156 \times 10^{-6}T\frac{\partial T}{\partial x}\right)sp + (2.2838 \times 10^{-3} - 1.0981 \times 10^{-5}T - 1.6078 \times 10^{-6}T^2)\frac{\partial s}{\partial x}p + (2.2838 \times 10^{-3} - 1.0981 \times 10^{-5}T - 1.6078 \times 10^{-6}T^2)s\frac{\partial p}{\partial x} \quad (44\text{-b})$$

In addition:

$$\frac{\partial}{\partial x}\left(Js^{3/2}p\right) = \frac{\partial J}{\partial x}s^{3/2}p + J\frac{\partial}{\partial x}\left(s^{3/2}\right)p + Js^{3/2}\frac{\partial p}{\partial x} = 2.866125 \times 10^{-4}s^{1/2}\frac{\partial s}{\partial x}p + (1.91075 \times 10^{-4})s^{3/2}\frac{\partial p}{\partial x} \quad (45\text{-b})$$

And:

$$\frac{\partial}{\partial x}(Mp^2) = \left(-6.12293 \times 10^{-6}\frac{\partial T}{\partial x} + 1.05574 \times 10^{-7}T\frac{\partial T}{\partial x}\right)p^2 + 2.0 \times (8.50935 \times 10^{-5} - 6.12293 \times 10^{-6}T + 5.2787 \times 10^{-8}T^2)p\frac{\partial p}{\partial x} \quad (47\text{-b})$$

Also:

$$\frac{\partial}{\partial x}(Nsp^2) = \left(2.0816 \times 10^{-8}\frac{\partial T}{\partial x} + 1.83394 \times 10^{-9}T\frac{\partial T}{\partial x}\right)sp^2 + (-9.9348 \times 10^{-7} + 2.0816 \times 10^{-8}T + 9.1697 \times 10^{-10}T^2)\frac{\partial s}{\partial x}p^2 + 2.0 \times (-9.9348 \times 10^{-7} + 2.0816 \times 10^{-8}T + 9.1697 \times 10^{-10}T^2)sp\frac{\partial p}{\partial x} \quad (49\text{-b})$$

According to equations (35-b). (36-b). (38-b). (40-b). (42-b). (44-b). (45-b). (47-b) and (49-b):

$$\frac{\partial}{\partial x}[K_T(s.T.p)] = (36\text{-b}) + (38\text{-b}) + (40\text{-b}) + (42\text{-b}) + (44\text{-b}) + (45\text{-b}) + (47\text{-b}) + (49\text{-b}). \quad (50\text{-b})$$

Furthermore. substituting (50-b) into equation (24-b) yields:

$$\frac{\partial}{\partial x}\left[1 - {}^{p}/_{K_T(s.T.p)}\right] = \frac{{}^{p}\frac{\partial}{\partial x}K_T(s.T.p) - \frac{\partial p}{\partial x}K_T(s.T.p)}{[K_T(s.T.p)]^2} = \frac{p(50\text{-b}) - \frac{\partial p}{\partial y}K_T(s.T.p)}{[K_T(s.T.p)]^2} \quad (51\text{-b})$$

Substituting (34-b) and (51-b) in (25-b) yields:

$$v_{D_{c_I}} = -\frac{g}{f}\int_{z_1}^{z_2}\frac{1}{\frac{\rho(s.T.0)}{1 - {}^{p}/_{K_T(s.T.p)}}} \times \frac{(34\text{-b})\left[1 - {}^{p}/_{K_T(s.T.p)}\right] - \rho(s.T.0)(51\text{-b})}{\left[1 - {}^{p}/_{K_T(s.T.p)}\right]^2}dz \quad (52\text{-b})$$

where in equation (52-b). $v_{D_{c_I}}$ is northward component of the first version of dense current. g is acceleration due to gravity. f is Coriolis parameter. z_1 is lower level of the oceanic layer. z_2 is higher level of the oceanic layer. $\rho(s.T.0)$ is density of surface ocean defined in (8). p is pressure of seawater at any point considered. $K_T(s.T.p)$ is mean bulk modulus defined in (9). (34-b) is equation with the same number and (51-b) is also

equation with the same number and z stands for vertical axis of Cartesian coordinates system.

2.1. First special case of dense current: Thermal Current

We know that baroclinity of seawater is depending on horizontal gradient of seawater density. But if:

$$s = constant \text{ or } \left(\frac{\partial s}{\partial x} = 0 \ \wedge \ \frac{\partial s}{\partial y} = 0\right) \qquad (53\text{-}1)$$

and

$$p = constant \text{ or } \left(\frac{\partial p}{\partial x} = 0 \ \wedge \ \frac{\partial p}{\partial y} = 0\right) \qquad (53\text{-}2)$$

then. baroclinity of seawater related only to the horizontal gradient of seawater temperature.

By existing conditions (53-1) and (53-2). horizontal gradient of seawater temperature causes to produce baroclinity of ambience solely.

In these conditions; i.e.. conditions (53-1) and (53-2); the variation of density in the horizontal direction is merely related to variation of seawater temperature in horizontal direction causing baroclinity of the ocean. In these circumstances. we define the vectorial difference of geostrophic current vector at upper level and geostrophic current vector at lower level of the oceanic layer as. *Thermal Current*. i.e.:

$$\mathbb{v}_{\mathrm{T_c}} \equiv \mathbb{v}_{g(z_2)} - \mathbb{v}_{g(z_1)} \qquad (1\text{-Tc})$$

where in Equation (1-Tc) $\mathbb{v}_{\mathrm{T_c}}$ stands for thermal current vector. $\mathbb{v}_{g(z_2)}$ is geostrophic current at upper level of the oceanic layer (at less depth). and $\mathbb{v}_{g(z_1)}$ is geostrophic current at lower level of the oceanic layer (at more depth) where z_2 refers to level with less depth and z_1 refers to level with more depth. i.e.. $z_1 < z_2$. [see figure 2]

According to definition (1-Tc). eastward and northward components of thermal current can be shown as following:

$$u_{\mathrm{T_c}} = u_{g(z_2)} - u_{g(z_1)} \qquad (2\text{-Tc-a})$$

and

$$v_{\mathrm{T_c}} = v_{g(z_2)} - v_{g(z_1)} \qquad (2\text{-Tc-b})$$

where in equation (2-Tc-a). $u_{\mathrm{T_c}}$ is eastward component of thermal current. $u_{g(z_2)}$ is eastward component of geostrophic current at upper level of the oceanic layer. $u_{g(z_1)}$ is eastward component of geostrophic current at lower level of the oceanic layer; and in equation (2-Tc-b) $v_{\mathrm{T_c}}$ is northward component of thermal current. $v_{g(z_2)}$ is northward component of geostrophic current at upper level of the oceanic layer. $v_{g(z_1)}$ stands for northward component of geostrophic current at lower level of the oceanic layer and $z_1 < z_2$.

2.1.1 The first version of thermal current

For deriving formulae for the eastward component of the first version of thermal current. we should omit terms including $\frac{\partial s}{\partial y}$ and $\frac{\partial p}{\partial y}$ in equations (30-a). (32-a) and (33-a) for first step by accepting conditions (53-1) and (53-2). Now. we have following equations for deriving some part of eastward component of the first version of thermal current:

$$\frac{\partial}{\partial y}A = 6.793952 \times 10^{-2}\frac{\partial T}{\partial y} - 1.819058 \times 10^{-2}T\frac{\partial T}{\partial y} + 3.005055 \times 10^{-4}T^2\frac{\partial T}{\partial y} - 4.480332 \times 10^{-4}T^3\frac{\partial T}{\partial y} + 3.268166 \times 10^{-8}T^4\frac{\partial T}{\partial y} \qquad (28\text{-T-a})$$

And:

$$\frac{\partial}{\partial y}(Bs) = \left(-4.0899 \times 10^{-3}\frac{\partial T}{\partial y} + 1.52876 \times 10^{-4}T\frac{\partial T}{\partial y} - 2.47401 \times 10^{-6}T^2\frac{\partial T}{\partial y} + 2.155 \times 10^{-8}T^3\frac{\partial T}{\partial y}\right)s \qquad (30\text{-T-a})$$

Also:

$$\frac{\partial}{\partial y}\left(Cs^{3/2}\right) = \left(1.0227 \times 10^{-4}\frac{\partial T}{\partial y} - 3.3092 \times 10^{-6}T\frac{\partial T}{\partial y}\right)\left(s^{3/2}\right) \qquad (32\text{-T-a})$$

Therefore. derivative of seawater density at surface of ocean with respect to y. considering conditions (53-1) and (53-2); is organized by addition of equations (28-T-a). (30-T-a) and (32-T-a). i.e.:

$$\frac{\partial}{\partial y}\rho(s.T.0) = (28\text{-T-a}) + (30\text{-T-a}) + (32\text{-T-a}) = \rho_{0_{\mathrm{T_a}}} \qquad (34\text{-T-a})$$

In equation (34-T-a). the $\rho_{0_{\mathrm{T_a}}}$ is derivative of surface density of seawater with respect to y after imposing conditions (53-1) and (53-2) and subscript "T" in equations' number refers to "Thermal".

Likewise. omitting terms including $\frac{\partial s}{\partial y}$ and $\frac{\partial p}{\partial y}$ in equations (38-a). (40-a). (42-a). (44-a). (45-a). (47-a) and (49-a) by accepting conditions (53-1) and (53-2) is necessary. And we have remained following equations for deriving eastward component of the first version of thermal current:

$$\frac{\partial}{\partial y}(E) = \frac{\partial}{\partial y}(1.965221 \times 10^4 + 1.484206 \times 10^2T - 2.327105T^2 + 1.360477 \times 10^{-2}T^3 - 5.155288 \times 10^{-5}T^4) = 1.484206 \times 10^2\frac{\partial T}{\partial y} - 4.65421T\frac{\partial T}{\partial y} + 4.081431 \times 10^{-2}T^2\frac{\partial T}{\partial y} - 2.0621152 \times 10^{-4}T^3\frac{\partial T}{\partial y} \qquad (36\text{-T-a})$$

And:

$$\frac{\partial}{\partial y}(Fs) = \left(-6.03459 \times 10^{-1}\frac{\partial T}{\partial y} + 2.19974 \times 10^{-2}T\frac{\partial T}{\partial y} - 1.8501 \times 10^{-4}T^2\frac{\partial T}{\partial y}\right)s \quad (38\text{-T-a})$$

In addition:

$$\frac{\partial}{\partial y}\left(Gs^{3/2}\right) = \left(1.6483 \times 10^{-2}\frac{\partial T}{\partial y} - 1.06018 \times 10^{-3}T\frac{\partial T}{\partial y}\right)s^{3/2} \quad (40\text{-T-a})$$

Also:

$$\frac{\partial}{\partial y}(Hp) = \left(1.43713 \times 10^{-3}\frac{\partial T}{\partial y} + 2.32184 \times 10^{-4}T\frac{\partial T}{\partial y} - 1.733715 \times 10^{-6}T^2\frac{\partial T}{\partial y}\right)p \quad (42\text{-T-a})$$

And:

$$\frac{\partial}{\partial y}(Isp) = \left(-1.0981 \times 10^{-5}\frac{\partial T}{\partial y} - 3.2156 \times 10^{-6}T\frac{\partial T}{\partial y}\right)sp \quad (44\text{-T-a})$$

In addition:

$$\frac{\partial}{\partial y}(Mp^2) = \left(-6.12293 \times 10^{-6}\frac{\partial T}{\partial y} + 1.05574 \times 10^{-7}T\frac{\partial T}{\partial y}\right)p^2 \quad (47\text{-T-a})$$

Also:

$$\frac{\partial}{\partial y}(Nsp^2) = \left(2.0816 \times 10^{-8}\frac{\partial T}{\partial y} + 1.83394 \times 10^{-9}T\frac{\partial T}{\partial y}\right)sp^2 \quad (49\text{-T-a})$$

Equations (28-T-a). (30-T-a). (32-T-a). (36-T-a). (38-T-a). (40-T-a). (42-T-a). (44-T-a). (47-T-a) and (49-T-a); are equations those prepared by imposing conditions (53-1) and (53-2) for eastward component of the first version of thermal current.

Now. according to equations. (36-T-a). (38-T-a). (40-T-a). (42-T-a). (44-T-a). (47-T-a) and (49-T-a) plus imposing conditions (53-1) and (53-2); equation (50-a) will be:

$$\frac{\partial}{\partial y}[K_T(s.T.p)] = \text{(36-T-a) + (38-T-a) + (40-T-a) + (42-T-a) + (44-T-a) + (47-T-a) + (49-T-a).} \quad (50\text{-T-a})$$

Imposing conditions (53-1) and (53-2) into (24-a). we get:

$$\frac{\partial}{\partial y}\left[1 - {}^{p}/_{K_T(s.T.p)}\right] = \frac{p\frac{\partial}{\partial y}K_T(s.T.p)}{[K_T(s.T.p)]^2} \quad (24\text{-T-a})$$

Substituting (50-T-a) into (24-T-a) has following result:

$$\frac{\partial}{\partial y}\left[1 - {}^{p}/_{K_T(s.T.p)}\right] = \frac{p\,(50\text{-T-a})}{[K_T(s.T.p)]^2} \quad (51\text{-T-a})$$

And substituting (34-T-a) and (51-T-a) in equation (23-a); modify it from eastward component of the first version of dense current. to eastward component of the first version of thermal current. i.e.:

$$u_{T_{c_I}} = \frac{g}{f}\int_{z_1}^{z_2}\frac{\frac{1}{\rho(s.T.0)}}{1-{}^{p}/_{K_T(s.T.p)}} \times \frac{(34\text{-T-a})\left[1-{}^{p}/_{K_T(s.T.p)}\right]-\rho(s.T.0)(51\text{-T-a})}{\left[1-{}^{p}/_{K_T(s.T.p)}\right]^2}dz \quad (5\text{-Tc-I-a})$$

In equation (5-Tc-I-a). $u_{T_{c_I}}$ is eastward component of the first version of thermal current. g is acceleration due to gravity. f is Coriolis parameter. z_1 is depth of lower level of the oceanic layer. z_2 is upper level of the oceanic layer. $\rho(s.T.0)$ is density of surface ocean defined in (8). p is pressure of water at any point considered. $K_T(s.T.p)$ is mean bulk modulus defined in (9). (34-T-a) is equation with the same number and (51-T-a) is also equation with the same number.

And for deriving formulae for the northward component of the first version of thermal current. we should omit terms including $\frac{\partial s}{\partial x}$ and $\frac{\partial p}{\partial x}$ in equations (30-b). (32-b) and (33-b) for first step by accepting conditions (53-1) and (53-2). Now. we have following equations for deriving some part of northward component of the first version of thermal current:

$$\frac{\partial}{\partial x}A = 6.793952 \times 10^{-2}\frac{\partial T}{\partial x} - 1.819058 \times 10^{-2}T\frac{\partial T}{\partial x} + 3.005055 \times 10^{-4}T^2\frac{\partial T}{\partial x} - 4.480332 \times 10^{-4}T^3\frac{\partial T}{\partial x} + 3.268166 \times 10^{-8}T^4\frac{\partial T}{\partial x} \quad (28\text{-T-b})$$

And:

$$\frac{\partial}{\partial x}(Bs) = \left(-4.0899 \times 10^{-3}\frac{\partial T}{\partial x} + 1.52876 \times 10^{-4}T\frac{\partial T}{\partial x} - 2.47401 \times 10^{-6}T^2\frac{\partial T}{\partial x} + 2.155 \times 10^{-8}T^3\frac{\partial T}{\partial x}\right)s \quad (30\text{-T-b})$$

Also:

$$\frac{\partial}{\partial x}\left(Cs^{3/2}\right) = \left(1.0227 \times 10^{-4}\frac{\partial T}{\partial x} - 3.3092 \times 10^{-6}T\frac{\partial T}{\partial x}\right)\left(s^{3/2}\right) \quad (32\text{-T-b})$$

Therefore. derivative of seawater density at surface of ocean with respect to x. considering conditions (53-1) and (53-2); is organized by addition of equations (28-T-b). (30-T-b) and (32-T-b). i.e.:

$$\frac{\partial}{\partial x}\rho(s.T.0) = \text{(28-T-b) + (30-T-b) + (32-T-b)} = \rho_{0_{T_b}}. \quad (34\text{-T-b})$$

In equation (34-T-b). the $\rho_{0_{T_b}}$ is derivative of surface density of seawater with respect to x after imposing

conditions (53-1) and (53-2) and subscript "T" in equations' number refers to "Thermal".

Likewise. omitting terms including $\frac{\partial s}{\partial x}$ and $\frac{\partial p}{\partial x}$ in equations (38-b). (40-b). (42-b). (44-b). (45-b). (47-b) and (49-b) by accepting conditions (53-1) and (53-2) is necessary. And we have remained following equations for deriving northward component of the first version of thermal current:

$$\frac{\partial}{\partial x}(E) = \frac{\partial}{\partial x}(1.965221 \times 10^4 + 1.484206 \times 10^2 T - 2.327105T^2 + 1.360477 \times 10^{-2}T^3 - 5.155288 \times 10^{-5}T^4) = 1.484206 \times 10^2 \frac{\partial T}{\partial x} - 4.65421T\frac{\partial T}{\partial x} + 4.081431 \times 10^{-2}T^2\frac{\partial T}{\partial x} - 2.0621152 \times 10^{-4}T^3\frac{\partial T}{\partial x} \quad \text{(36-T-b)}$$

And:

$$\frac{\partial}{\partial x}(Fs) = \left(-6.03459 \times 10^{-1}\frac{\partial T}{\partial x} + 2.19974 \times 10^{-2}T\frac{\partial T}{\partial x} - 1.8501 \times 10^{-4}T^2\frac{\partial T}{\partial x}\right)s \quad \text{(38-T-b)}$$

In addition:

$$\frac{\partial}{\partial x}\left(Gs^{3/2}\right) = \left(1.6483 \times 10^{-2}\frac{\partial T}{\partial x} - 1.06018 \times 10^{-3}T\frac{\partial T}{\partial x}\right)s^{3/2} \quad \text{(40-T-b)}$$

Also:

$$\frac{\partial}{\partial x}(Hp) = \left(1.43713 \times 10^{-3}\frac{\partial T}{\partial x} + 2.32184 \times 10^{-4}T\frac{\partial T}{\partial x} - 1.733715 \times 10^{-6}T^2\frac{\partial T}{\partial x}\right)p \quad \text{(42-T-b)}$$

And:

$$\frac{\partial}{\partial x}(Isp) = \left(-1.0981 \times 10^{-5}\frac{\partial T}{\partial x} - 3.2156 \times 10^{-6}T\frac{\partial T}{\partial x}\right)sp \quad \text{(44-T-b)}$$

In addition:

$$\frac{\partial}{\partial x}(Mp^2) = \left(-6.12293 \times 10^{-6}\frac{\partial T}{\partial x} + 1.05574 \times 10^{-7}T\frac{\partial T}{\partial x}\right)p^2 \quad \text{(47-T-b)}$$

Also:

$$\frac{\partial}{\partial x}(Nsp^2) = \left(2.0816 \times 10^{-8}\frac{\partial T}{\partial x} + 1.83394 \times 10^{-9}T\frac{\partial T}{\partial x}\right)sp^2 \quad \text{(49-T-b)}$$

Equations (28-T-b). (30-T-b). (32-T-b). (36-T-b). (38-T-b). (40-T-b). (42-T-b). (44-T-b). (47-T-b) and (49-T-b); are equations those prepared by imposing conditions (53-1) and (53-2) for northward component of the first version of thermal current.

Now. according to equations. (36-T-b). (38-T-b). (40-T-b). (42-T-b). (44-T-b). (47-T-b) and (49-T-b) plus imposing conditions (53-1) and (53-2); equation (50-b) will be:

$$\frac{\partial}{\partial x}[K_T(s.T.p)] = \text{(36-T-b) + (38-T-b) + (40-T-b) + (42-T-b) + (44-T-b) + (47-T-b) + (49-T-b).} \quad \text{(50-T-b)}$$

By imposing conditions (53-1) and (53-2) into (24-b). we get:

$$\frac{\partial}{\partial x}\left[1 - {}^{p}/_{K_T(s.T.p)}\right] = \frac{p\frac{\partial}{\partial x}K_T(s.T.p)}{[K_T(s.T.p)]^2} \quad \text{(24-T-b)}$$

Substituting (50-T-b) into (24-T-b) has following result:

$$\frac{\partial}{\partial x}\left[1 - {}^{p}/_{K_T(s.T.p)}\right] = \frac{p\ (50-\text{T}-\text{b})}{[K_T(s.T.p)]^2} \quad \text{(51-T-b)}$$

And substituting (34-T-b) and (51-T-b) in equation (23-b); modify it from northward component of the first version of dense current. to northward component of the first version of thermal current. i.e.:

$$v_{\text{T}_{\text{c}_\text{I}}} = -\frac{g}{f}\int_{z_1}^{z_2} \frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}} \times \frac{(34-\text{T}-\text{b})\left[1-{}^{p}/_{K_T(s.T.p)}\right]-\rho(s.T.0)(51-\text{T}-\text{b})}{\left[1-{}^{p}/_{K_T(s.T.p)}\right]^2}dz \quad \text{(5-Tc-I-b)}$$

In equation (5-Tc-I-b). $v_{\text{T}_{\text{c}_\text{I}}}$ is northward component of the first version of thermal current. g is acceleration due to gravity. f is Coriolis parameter. z_1 is depth of lower level of the oceanic layer. z_2 is upper level of the oceanic layer. $\rho(s.T.0)$ is density of surface ocean defined in (8). p is pressure of seawater at any point considered. $K_T(s.T.p)$ is mean bulk modulus defined in (9). (34-T-b) is equation with the same number and (51-T-b) is also equation with the same number.

Also. considering conditions (53-1) and (53-2). then combining equations (22-a) and (22-b) yields:

$$\mathbb{v}_{\text{T}_{\text{c}_\text{I}}} = -\frac{g}{f}\int_{z_1}^{z_2} \frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}} \mathbb{k} \times \nabla\left[\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}\right]dz \quad \text{(4-Tc-I)}$$

Equation (4-Tc-I) is *The first version of thermal current vector* where in it. $\mathbb{v}_{\text{T}_{\text{c}_\text{I}}}$ is the first version of thermal current vector. g is acceleration due to gravity. f is Coriolis parameter. z_1 is depth of lower level of the oceanic layer. z_2 is upper level of the oceanic layer. $\rho(s.T.0)$ is density of surface ocean defined in (8). p is pressure of seawater at any point considered. $K_T(s.T.p)$ is mean bulk modulus defined in (9). $\mathbb{k}$ is vertical unit vector in Cartesian coordinates system. ∇ is gradient operator in Cartesian coordinates system and z is vertical axis in Cartesian coordinates system.

Eastward component of the first version of thermal current vector is introduced in equation (5-Tc-I-a) and northward component of it. is presented in equation

(5-Tc-I-b). Furthermore. in equation (5-Tc-I-b). derivative of seawater density with respect to x and in equation (5-Tc-I-a) derivative of seawater density with respect to y have done considering conditions (53-1) and (53-2)[4]

From the first version of thermal current vector or its components. one can perceive:

2 – 1 – 1 – A: Whatever we go from pole to equator. the first version of thermal current becomes stronger (with pay attention to footnote No. 2)

2 – 1 – 1 – B: If the oceanic layer has more thickness; then the first version of the thermal current becomes more powerful;

2 – 1 – 1 – C: Whatever oceanic layer is near to ocean surface. density would be lower and the first version of the thermal current becomes stronger as a result;

2 – 1 – 1 – D: if the temperature of the oceanic layer would be high – due to decreasing seawater density – the first version of thermal current is more powerful and

2 – 1 – 1 – E: If the horizontal gradient of seawater temperature would be greater. the first version of thermal current becomes more powerful. because the first version of the thermal current is proportional to the horizontal gradient of implicit seawater temperature.

In this manner. the first version of thermal current vector. i.e.. equation (4-Tc-I) shows that: "*The first version of thermal current flows parallel to Isopycnals and Isopleths of implicit seawater temperature so that. low values of density (high values of temperature) are located at the right side of downstream.*" (In the northern hemisphere) This fact is demonstrated in figures 3 and 4.

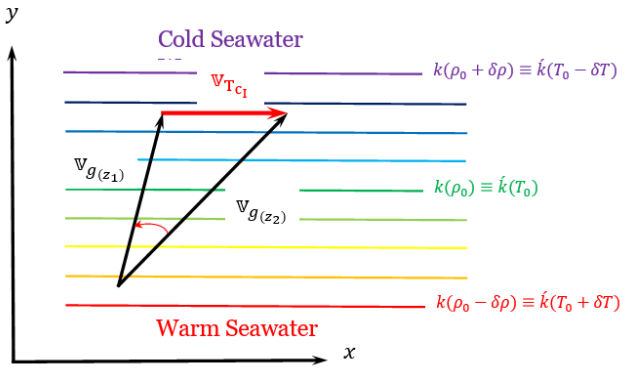

Figure 3. Counterclockwise rotation of geostrophic current with respect to depth (Backing) and warm seawater advection.

Coefficient $k=-\frac{g}{f}\langle\frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}}\rangle(z_2-z_1)$ and $\acute{k}$ is proportional to k in such a manner that – for instance – $k(\rho_0-\delta\rho)$ is identical to $\acute{k}(T_0+\delta T)$. [The first version of thermal current]

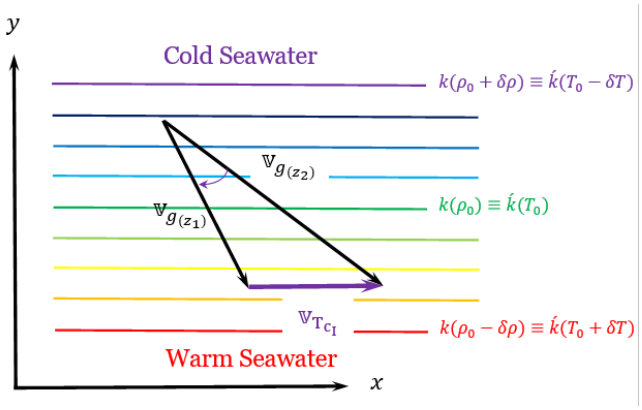

Figure 4. Clockwise rotation of geostrophic current with respect to depth (Veering) and cold seawater advection. Coefficient $k=-\frac{g}{f}\langle\frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}}\rangle(z_2-z_1)$ and $\acute{k}$ is proportional to k in such a manner that – for instance – $k(\rho_0+\delta\rho)$ is identical to $\acute{k}(T_0-\delta T)$. [The first version of thermal current]

In connection with figure 3. counterclockwise turning of geostrophic current with respect to depth (backing) is associated with warmer seawater advection by geostrophic current in the layer.

Conversely; as shown in figure 4. clockwise turning of geostrophic current with respect to depth (veering) implies advection of colder seawater by geostrophic current in the oceanic layer. Also $\langle\frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}}\rangle$ under explanations of figures 3 and 4 is vertical average of specific volume of seawater in the oceanic layer[5].

2.1.2 The second version of thermal current

Regarding to conditions (53-1) and (53-2). we had:

$$u_{T_{c_I}}=\frac{g}{f}\int_{z_1}^{z_2}\frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}}\times\frac{(34-\mathrm{T}-\mathrm{a})\left[1-{}^{p}/_{K_T(s.T.p)}\right]-\rho(s.T.0)(51-\mathrm{T}-\mathrm{a})}{\left[1-{}^{p}/_{K_T(s.T.p)}\right]^2}dz \quad \text{(5-Tc-I-a)}$$

Equation (5-Tc-I-a) was eastward component of the first version of thermal current.

[4] Note that limit ${}^{p}/_{K_T(s.T.p)}$ when it goes to zero from right hand is near but greater than zero. Therefore. for every p and every $K_T(s.T.p)$. we have:

$$0<1-{}^{p}/_{K_T(s.T.p)}\leq 1$$

[5] There are many procedures about averaging in meteorology and subbranches. Every method for averaging particular parameter or term. is related to position. time and specifications of the domain. All meteorological or oceanographical parameters; decreases logarithmic or semi-logarithmic and increases exponentially or semi-exponentially with respect to height or depth. Therefore. for every case. we need to use special manner for averaging. As a simple example. if we consider a layer of seawater between 200- and 300-meters depth; the easiest way is using linear averaging. i.e.. adding the value of parameters or terms of above and below levels of the oceanic layer and dividing the result by 2.

In almost all meteorological purposes. we regard acceleration due to gravity as constant. Consequently. we can enter it inside integral sign. considering conditions (53-1) and (53-2); to get eastward component of the second version of thermal current i.e.:

$$u_{T_{c_{II}}} = \frac{1}{f}\int_{z_1}^{z_2} \frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}} \times \frac{(34-\mathrm{T}-\mathrm{a})\left[1-{}^{p}/_{K_T(s.T.p)}\right]-\rho(s.T.0)(51-\mathrm{T}-\mathrm{a})}{\left[1-{}^{p}/_{K_T(s.T.p)}\right]^2} d\Phi_{\mathrm{T}} \quad \text{(5-Tc-II-a)}$$

where in equation (5-Tc-II-a). $u_{T_{c_{II}}}$ is eastward component of the second version of thermal current. f is Coriolis parameter. $z_1$is lower level of the oceanic layer. z_2 is higher level of the oceanic layer. $\rho(s.T.0)$ is density of surface ocean defined in (8). p is pressure of seawater at any point considered. $K_T(s.T.p)$ is mean bulk modulus defined in (9). (34-T-a) is equation with the same number and (51-T-a) is also equation with the same number. Φ is geopotential and subscript "T" refers to this fact that variation of seawater geopotential is affiliated to seawater temperature solely. with due attention to: [3]

$$\Phi = \Phi(s.T.p) \quad (35)$$

And for deriving formula for the northward component of the second version of thermal current. by considering conditions (53-1) and (53-2); we fix the mind on equation (5-Tc-I-b) that was northward component of the first version of thermal current i.e.:

$$v_{T_{c_I}} = -\frac{g}{f}\int_{z_1}^{z_2} \frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}} \times \frac{(34-\mathrm{T}-\mathrm{b})\left[1-{}^{p}/_{K_T(s.T.p)}\right]-\rho(s.T.0)(51-\mathrm{T}-\mathrm{b})}{\left[1-{}^{p}/_{K_T(s.T.p)}\right]^2} dz \quad \text{(5-Tc-I-b)}$$

Equation (5-Tc-I-b) was northward component of the first version of thermal current. and we can enter acceleration due to gravity inside integral sign to get northward component of the second version of thermal current. i.e.:

$$v_{T_{c_{II}}} = -\frac{1}{f}\int_{z_1}^{z_2} \frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}} \times \frac{(34-\mathrm{T}-\mathrm{b})\left[1-{}^{p}/_{K_T(s.T.p)}\right]-\rho(s.T.0)(51-\mathrm{T}-\mathrm{b})}{\left[1-{}^{p}/_{K_T(s.T.p)}\right]^2} d\Phi_{\mathrm{T}} \quad \text{(5-Tc-II-b)}$$

where in equation (5-Tc-II-b). $v_{T_{c_{II}}}$ is northward component of the second version of thermal current. f is Coriolis parameter. $z_1$is lower level of the oceanic layer. z_2 is higher level of the oceanic layer. $\rho(s.T.0)$ is density of surface ocean defined in (8). p is pressure of seawater at any point considered. $K_T(s.T.p)$ is mean bulk modulus defined in (9). (34-T-b) is equation with the same number and (51-T-b) is also equation with the same number. Φ is geopotential and subscript "T" refers to this fact that variation of seawater geopotential is affiliated to seawater temperature solely. with due attention to (35)

The first version of thermal current vector was:

$$\mathbb{v}_{T_{c_I}} = -\frac{g}{f}\int_{z_1}^{z_2} \frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}} \mathbb{k} \times \nabla\left[\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}\right] dz \quad \text{(4-Tc-I)}$$

If we enter acceleration due to gravity inside integral sign. we get the second version of thermal current: i.e.:

$$\mathbb{v}_{T_{c_{II}}} = -\frac{1}{f}\int_{z_1}^{z_2} \frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}} \mathbb{k} \times \nabla\left[\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}\right] d\Phi_{\mathrm{T}} \quad \text{(4-Tc-II)}$$

Equation (4-Tc-II) is <u>*The second version of thermal current vector*</u> where in it. $\mathbb{v}_{T_{c_{II}}}$ is the second version of thermal current vector. f is Coriolis parameter. z_1 is depth of lower level of the oceanic layer. z_2 is upper level of the oceanic layer. $\rho(s.T.0)$ is density of surface ocean defined in (8). p is pressure of seawater at any point considered. $K_T(s.T.p)$ is mean bulk modulus defined in (9). $\mathbb{k}$ is vertical unit vector in Cartesian coordinates system. ∇ is gradient operator in Cartesian coordinates system. Φ is geopotential and subscript " T " refers to this fact that variation of seawater geopotential is affiliated to seawater temperature solely.

Eastward component of the second version of thermal current vector is introduced in equation (5-Tc-II-a) and northward component of it. is presented in equation (5-Tc-II-b).

From the second version of thermal current vector or its components. one can perceive:

2 – 1 – 2 – A: Whatever we go from pole to equator. the second version of thermal current becomes stronger (with pay attention to footnote No. 2)

2 – 1 – 2 – B: If the oceanic layer has more geopotential thickness; then the second version of the thermal current becomes more powerful;

2 – 1 – 2 – C: Whatever oceanic layer is near to ocean surface; density would be lower and the second version of the thermal current becomes stronger as a result;

2 – 1 – 2 – D: if the temperature of the oceanic layer would be high. the second version of thermal current is more powerful and

2 – 1 – 2 – E: If the horizontal gradient of seawater temperature would be greater. the second version of thermal current becomes more powerful. because the second version of the thermal current is proportional to the horizontal gradient of implicit seawater temperature.

In this manner. the second version of thermal current vector. i.e.. equation (4-Tc-II) shows that: "*The second version of thermal current flows parallel to Isopycnals and Isopleths of implicit seawater temperature as well as isopleths of geopotentials so that. low values of density (high values of temperature or high values of geopotentials) are located at the right side of downstream.*" (In the northern hemisphere) This fact is illustrated in figures 5 and 6.

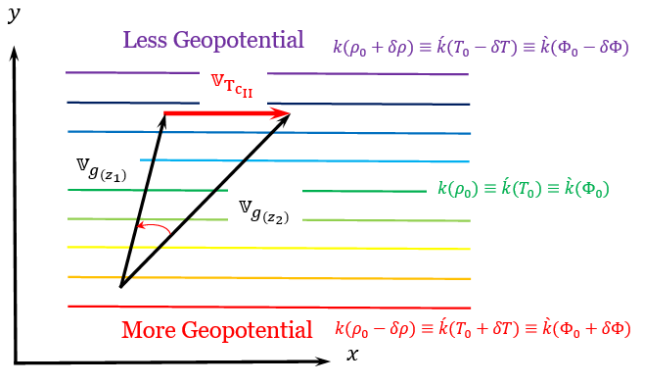

Figure 5. Counterclockwise rotation of geostrophic current with respect to depth (Backing) and more geopotential advection. Coefficient $k=-\frac{1}{f}\langle\frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}}\rangle(\Phi_2-\Phi_1)$. $\acute{k}$ is proportional to k in such a manner that – for instance – $k(\rho_0-\delta\rho)$ is identical to $\acute{k}(T_0+\delta T)$ and $\grave{k}$ is proportional to k in style that – for example – $k(\rho_0-\delta\rho)$ is similar in very detail to $\grave{k}(\Phi_0+\delta\Phi)$ [The second version of thermal current]

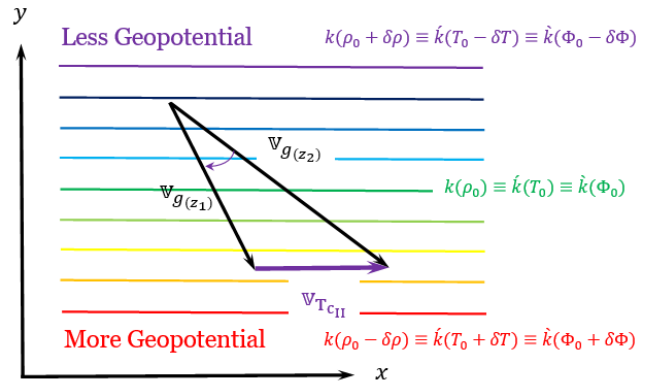

Figure 6. Clockwise rotation of geostrophic current with respect to depth (Veering) and less geopotential advection. Coefficient $k=-\frac{1}{f}\langle\frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}}\rangle(\Phi_2-\Phi_1)$. $\acute{k}$ is proportional to k in such a manner that – for instance – $k(\rho_0+\delta\rho)$ is identical to $\acute{k}(T_0-\delta T)$ and $\grave{k}$ is proportional to k in style that – for example – $k(\rho_0+\delta\rho)$ is similar in very detail to $\grave{k}(\Phi_0-\delta\Phi)$ [The second version of thermal current]

In connection with figure 5 – considering conditions (53-1) and (53-2) – counterclockwise turning of geostrophic current with respect to depth (backing) is associated with more geopotential advection in the ocean by geostrophic current in the layer.
Conversely; as shown in figure 6 – considering conditions (53-1) and (53-2) – clockwise turning of geostrophic current with respect to depth (veering) implies less geopotential advection in the ocean by geostrophic current in the oceanic layer.

2.1.3 The third version of thermal current

For deriving formulae for the third version of thermal current by considering conditions (53-1). (53-2) and this fact that. variation of geopotential is related to variation of seawater temperature only; first we fix the mind on equation (5-Dc-III-a) that was eastward components of the third version of dense current i.e.:

$$u_{D_{c_{III}}}=-\frac{1}{f}\frac{\partial}{\partial y}(\Phi_2-\Phi_1) \qquad \text{(5-Dc-III-a)}$$

In equation (5-Dc-III-a). $u_{D_{c_{III}}}$ is eastward component of the third version of dense current and we used definition (2-Dc-a) for deriving this equation.
And northward component of the third version of dense current was:

$$v_{D_{c_{III}}}=\frac{1}{f}\frac{\partial}{\partial x}(\Phi_2-\Phi_1) \qquad \text{(5-Dc-III-b)}$$

In equation (5-Dc-III-b). $v_{D_{c_{III}}}$ is northward component of the third version of dense current and we used definition (2-Dc-b) for extracting this equation.
With due attention to: [3]

$$\Phi=\Phi(s.T.p) \qquad (35)$$

and emphasis on conditions (53-1) and (53-2) based upon that horizontal variation of seawater density is related to horizontal variation of seawater temperature solely. and implies variation of seawater geopotential height only by horizontal variation of seawater temperature. we get:

$$u_{T_{c_{III}}}=-\frac{1}{f}\frac{\partial}{\partial y}\left(\Phi_{T_2}-\Phi_{T_1}\right) \qquad \text{(5-Tc-III-a)}$$

where. in equation (5-Tc-III-a). $u_{T_{c_{III}}}$ is eastward component of the third version of thermal current. f is Coriolis parameter. y is northward axis of Cartesian coordinates system. Φ_{T_2} is geopotential of higher level of the oceanic layer. Φ_{T_1} is geopotential of lower level of the oceanic layer and $\left(\Phi_{T_2}-\Phi_{T_1}\right)$ is geopotential thickness of the oceanic layer. Furthermore. subscript "T" refers to this fact that increasing or decreasing of geopotential thickness is related on seawater temperature only. with due attention to (35).
Also:

$$v_{T_{c_{III}}}=\frac{1}{f}\frac{\partial}{\partial x}\left(\Phi_{T_2}-\Phi_{T_1}\right) \qquad \text{(5-Tc-III-b)}$$

where. in equation (5-Tc-III-b). $v_{T_{c_{III}}}$ is northward component of the third version of thermal current. f is Coriolis parameter. x is eastward axis of Cartesian coordinates system. Φ_{T_2} is geopotential of higher level

of the oceanic layer. Φ_{T_1} is geopotential of lower level of the oceanic layer and $(\Phi_{T_2} - \Phi_{T_1})$ is geopotential thickness of the oceanic layer. Furthermore. subscript "T" refers to this fact that increasing or decreasing of geopotential thickness is related on seawater temperature only. with due attention to (35).
In addition. by joining (5-Tc-III-a) and (5-Tc-III-b) as vector. we obtain:

$$\mathbb{v}_{T_{c_{III}}} = f^{-1}\mathbb{k} \times \nabla_p(\Phi_{T_2} - \Phi_{T_1}) \qquad (4\text{-Tc-III})$$

where. in equation (4-Tc-III). $\mathbb{v}_{T_{c_{III}}}$ is: *The third version of thermal current vector*. f is Coriolis parameter. $\mathbb{k}$ is vertical unit vector in pressure coordinates system. ∇_p is gradient operator in pressure coordinates system. Φ_{T_2} is geopotential of higher level of the oceanic layer. Φ_{T_1} is geopotential of lower level of the oceanic layer and $(\Phi_{T_2} - \Phi_{T_1})$ is geopotential thickness of the oceanic layer. Furthermore. subscript "T" refers to this fact that increasing or decreasing of geopotential is related on seawater temperature only. with due attention to (35).
From the third version of thermal current vector or its components. one can perceive:
2 – 1 – 3 – A: Whatever we go from pole to equator. the third version of thermal current becomes stronger (with pay attention to footnote No. 2)
2 – 1 – 3 – B: If the oceanic layer has more geopotential thickness; then the third version of the thermal current becomes more powerful. and
2 – 1 – 3 – C: If the horizontal gradient of seawater geopotential thickness would be greater. the third version of thermal current becomes more powerful. because the third version of the thermal current is proportional to the horizontal gradient of seawater geopotential thickness.
In this manner. the third version of thermal current vector. i.e.. equation (4-Tc-III) shows that: "*The third version of thermal current flows parallel to Isopleths of geopotential thickness of the oceanic layer* i.e.. $(\Phi_{T_2} - \Phi_{T_1})$ *so that. high values of geopotential thickness are located at the right side of downstream.*" (In the northern hemisphere) This fact is demonstrated in figures 7 and 8.

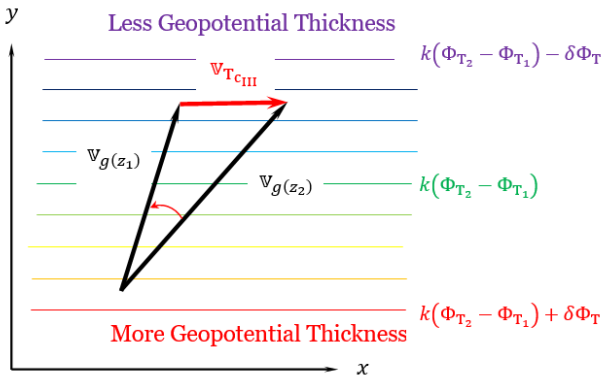

Figure 7. Counterclockwise rotation of geostrophic current with respect to depth (Backing) and more geopotential thickness advection. Coefficient $k = \frac{1}{f}$ [The third version of thermal current]

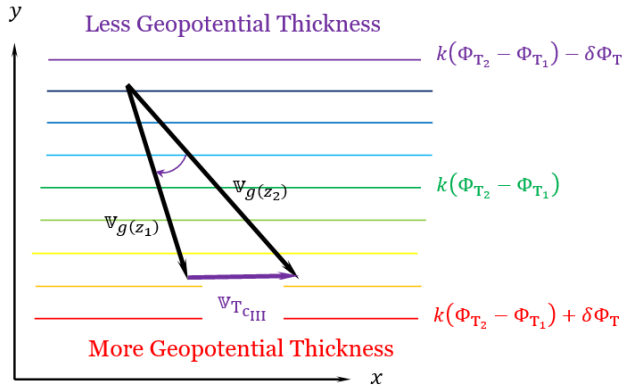

Figure 8. Clockwise rotation of geostrophic current with respect to depth (Veering) and less geopotential thickness advection. Coefficient $k = \frac{1}{f}$ [The third version of thermal current]

In connection with figure 7 – considering conditions (53-1) and (53-2) – counterclockwise turning of geostrophic current with respect to depth (backing) is associated with more geopotential thickness advection of the oceanic layer by geostrophic current in the layer. Conversely; as shown in figure 8 – considering conditions (53-1) and (53-2) – clockwise turning of geostrophic current with respect to depth (veering) implies less geopotential thickness advection of oceanic layer by geostrophic current in the oceanic layer.

2.2. Second special case of dense current: Saline Current

As we memorize; baroclinity of seawater is depending on horizontal gradient of seawater density. But if:

$$p = constant \text{ or } \left(\frac{\partial p}{\partial x} = 0 \ \wedge \ \frac{\partial p}{\partial y} = 0\right) \qquad (53\text{-}2)$$

and

$$T = constant \text{ or } \left(\frac{\partial T}{\partial x} = 0 \ \wedge \ \frac{\partial T}{\partial y} = 0\right) \qquad (53\text{-}3)$$

then. baroclinity of seawater related only to the horizontal gradient of seawater salinity.
By existing conditions (53-2) and (53-3). horizontal gradient of seawater salinity causes to produce baroclinity of ambience solely.
In these conditions; i.e.. conditions (53-2) and (53-3); the variation of density in the horizontal direction is merely related to variation of seawater salinity in horizontal direction causing baroclinity of the ocean. In these circumstances. we define the vectorial difference of geostrophic current vector at upper level and geostrophic current vector at lower level of the oceanic

layer as. *Saline Current*. (by pay attention to figure 2) i.e.:

$$\mathbb{v}_{S_c} \equiv \mathbb{v}_{g(z_2)} - \mathbb{v}_{g(z_1)} \qquad \text{(1-Sc)}$$

where in Equation (1-Sc) $\mathbb{v}_{S_c}$ stands for saline current vector. $\mathbb{v}_{g(z_2)}$ is geostrophic current at upper level of the oceanic layer (at less depth). and $\mathbb{v}_{g(z_1)}$ is geostrophic current at lower level of the oceanic layer (at more depth) where z_2 refers to level with less depth and z_1 refers to level with more depth. i.e.. $z_1 < z_2$. [see figure 2]

According to definition (1-Sc). eastward and northward components of saline current can be shown as following:

$$u_{S_c} = u_{g(z_2)} - u_{g(z_1)} \qquad \text{(2-Sc-a)}$$

and

$$v_{S_c} = v_{g(z_2)} - v_{g(z_1)} \qquad \text{(2-Sc-b)}$$

where in equation (2-Sc-a). u_{S_c} is eastward component of saline current. $u_{g(z_2)}$ is eastward component of geostrophic current at upper level of the oceanic layer. $u_{g(z_1)}$ is eastward component of geostrophic current at lower level of the oceanic layer; and in equation (2-Sc-b) v_{S_c} is northward component of saline current. $v_{g(z_2)}$ is northward component of geostrophic current at upper level of the oceanic layer. $v_{g(z_1)}$ stands for northward component of geostrophic current at lower level of the oceanic layer and $z_1 < z_2$.

2.2.1 The first version of saline current

For deriving formulae for the first version of saline current. first bear in mind equation (23-a) that was eastward component of the first version of dense current:

$$u_{D_{c_I}} = \frac{g}{f}\int_{z_1}^{z_2} \frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}} \times \frac{\frac{\partial\rho}{\partial y}(s.T.0)\left[1-{}^{p}/_{K_T(s.T.p)}\right] - \rho(s.T.0)\frac{\partial}{\partial y}\left[1-{}^{p}/_{K_T(s.T.p)}\right]}{\left[1-{}^{p}/_{K_T(s.T.p)}\right]^2} dz \qquad \text{(23-a)}$$

And we remind equations (26-a). (28-a). (30-a). (32-a). (33-a). (34-a). (35-a). (36-a). (38-a). (40-a). (42-a). (44-a). (45-a). (47-a). (49-a). (50-a). (51-a) and (52-a); and applying conditions (53-2) and (53-3) on them. The results are as following:

$$\frac{\partial}{\partial y}(Bs) = (8.24493 \times 10^{-1} - 4.0899 \times 10^{-3}T + 7.6438 \times 10^{-5}T^2 - 8.2467 \times 10^{-7}T^3 + 5.3875 \times 10^{-9}T^4)\frac{\partial s}{\partial y} \qquad \text{(30-s-a)}$$

And:

$$\frac{\partial}{\partial y}\left(Cs^{3}/_{2}\right) = (-5.72466 \times 10^{-3} + 1.0227 \times 10^{-4}T - 1.6546 \times 10^{-6}T^2)\left(\frac{3}{2}\, s^{1}/_{2}\frac{\partial s}{\partial y}\right) \qquad \text{(32-s-a)}$$

Also:

$$\frac{\partial}{\partial y}(Ds^2) = \frac{\partial}{\partial y}[(4.8314 \times 10^{-4})s^2] = 9.6628 \times 10^{-4} s\frac{\partial s}{\partial y} \qquad \text{(33-s-a)}$$

Therefore. derivative of seawater density at surface of ocean with respect to y. is organized by addition of equations (30-s-a). (32-s-a) and (33-s-a) and considering conditions (53-2) and (53-3). i.e.:

$$\frac{\partial}{\partial y}\rho(s.T.0) = \text{(30-s-a)} + \text{(32-s-a)} + \text{(33-s-a)} = \rho_{0_{s_a}} \qquad \text{(34-s-a)}$$

In equation (34-s-a). the $\rho_{0_{s_a}}$ is derivative of surface density of seawater with respect to y after imposing conditions (53-2) and (53-3) and subscript "s" in equations' number refers to "Saline".

Furthermore:

$$\frac{\partial}{\partial y}(Fs) = (5.46746 \times 10^{1} - 6.03459 \times 10^{-1}T + 1.09987 \times 10^{-2}T^2 - 6.1670 \times 10^{-5}T^3)\frac{\partial s}{\partial y} \qquad \text{(38-s-a)}$$

And:

$$\frac{\partial}{\partial y}\left(Gs^{3}/_{2}\right) = (7.944 \times 10^{-2} + 1.6483 \times 10^{-2}T - 5.3009 \times 10^{-4}T^2)\frac{3}{2}s^{1}/_{2}\frac{\partial s}{\partial y} \qquad \text{(40-s-a)}$$

In addition:

$$\frac{\partial}{\partial y}(Isp) = (2.2838 \times 10^{-3} - 1.0981 \times 10^{-5}T - 1.6078 \times 10^{-6}T^2)\frac{\partial s}{\partial y}p \qquad \text{(44-s-a)}$$

And with aid of equation (19):

$$\frac{\partial}{\partial y}\left(Js^{3}/_{2}p\right) = 2.866125 \times 10^{-4}s^{1}/_{2}\frac{\partial s}{\partial y}p \qquad \text{(45-s-a)}$$

Also:

$$\frac{\partial}{\partial y}(Nsp^2) = (-9.9348 \times 10^{-7} + 2.0816 \times 10^{-8}T + 9.1697 \times 10^{-10}T^2)\frac{\partial s}{\partial y}p^2 \qquad \text{(49-s-a)}$$

According to equations (35-a). (38-s-a). (40-s-a). (44-s-a). (45-s-a) and (49-s-a). we have:

$$\frac{\partial}{\partial y}[K_T(s.T.p)] = \text{(38-s-a)} + \text{(40-s-a)} + \text{(44-s-a)} + \text{(45-s-a)} + \text{(49-s-a)}. \qquad \text{(50-s-a)}$$

Contemplate conditions (53-2) and (53-3) and considering equation (50-s-a); equation (24-a) will be:

$$\frac{\partial}{\partial y}\left[1-{}^{p}/_{K_T(s.T.p)}\right]=\frac{p(50-\mathrm{s}-\mathrm{a})-\frac{\partial p}{\partial y}K_T(s.T.p)}{[K_T(s.T.p)]^2} \quad (51\text{-s-a})$$

Substituting (34-s-a) and (51-s-a) in (25-a) yields eastward component of the first version of the saline current. i.e.:

$$u_{S_{c_I}}=\frac{g}{f}\int_{z_1}^{z_2}\frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}}\times \frac{(34-\mathrm{s}-\mathrm{a})\left[1-{}^{p}/_{K_T(s.T.p)}\right]-\rho(s.T.0)(51-\mathrm{s}-\mathrm{a})}{\left[1-{}^{p}/_{K_T(s.T.p)}\right]^2}dz \quad (5\text{-Sc-I-a})$$

where in equation (5-Sc-I-a). $u_{S_{c_I}}$ is eastward component of the first version of saline current. g is acceleration due to gravity. f is Coriolis parameter. z_1 is lower level of the oceanic layer. z_2 is higher level of the oceanic layer. $\rho(s.T.0)$ is density of surface ocean defined in (8). p is pressure of seawater at any point considered. $K_T(s.T.p)$ is mean bulk modulus defined in (9). (34-s-a) is equation with the same number and (51-s-a) is also equation with the same number and z stands for vertical axis of Cartesian coordinates system.

And for deriving formulae for the northward component of the first version of saline current. first we fix the mind on equation (23-b) that was northward component of the first version of dense current:

$$v_{D_{c_I}}=-\frac{g}{f}\int_{z_1}^{z_2}\frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}}\times \frac{\frac{\partial}{\partial x}\rho(s.T.0)\left[1-{}^{p}/_{K_T(s.T.p)}\right]-\rho(s.T.0)\frac{\partial}{\partial x}\left[1-{}^{p}/_{K_T(s.T.p)}\right]}{\left[1-{}^{p}/_{K_T(s.T.p)}\right]^2}dz \quad (23\text{-b})$$

And we remind equations (26-b). (28-b). (30-b). (32-b). (33-b). (34-b). (35-b). (36-b). (38-b). (40-b). (42-b). (44-b). (45-b). (47-b). (49-b). (50-b). (51-b) and (52-b); and applying conditions (53-2) and (53-3) on them. The results are as following:

$$\frac{\partial}{\partial x}(Bs)=(8.24493\times10^{-1}-4.0899\times10^{-3}T+7.6438\times10^{-5}T^2-8.2467\times10^{-7}T^3+5.3875\times10^{-9}T^4)\frac{\partial s}{\partial x} \quad (30\text{-s-b})$$

And:

$$\frac{\partial}{\partial x}\left(Cs^{3}/_{2}\right)=(-5.72466\times10^{-3}+1.0227\times10^{-4}T-1.6546\times10^{-6}T^2)\left(\frac{3}{2}\,s^{1}/_{2}\frac{\partial s}{\partial x}\right) \quad (32\text{-s-b})$$

In addition:

$$\frac{\partial}{\partial x}(Ds^2)=\frac{\partial}{\partial x}[(4.8314\times10^{-4})s^2]=9.6628\times10^{-4}s\frac{\partial s}{\partial x} \quad (33\text{-s-b})$$

Therefore. derivative of seawater density at surface of ocean with respect to x. is organized by addition of equations (30-s-b). (32-s-b) and (33-s-b) considering conditions (53-2) and (53-3). i.e.:

$$\frac{\partial}{\partial x}\rho(s.T.0)=(30\text{-s-b})+(32\text{-s-b})+(33\text{-s-b})=\rho_{0_{s_b}} \quad (34\text{-s-b})$$

In equation (34-s-b). the $\rho_{0_{s_b}}$ is derivative of surface density of seawater with respect to x after imposing conditions (53-2) and (53-3) and subscript "s" in equations' number refers to "Saline".

Furthermore. contemplating conditions (53-2) and (53-3). we have:

$$\frac{\partial}{\partial x}(Fs)=(5.46746\times10^{1}-6.03459\times10^{-1}T+1.09987\times10^{-2}T^2-6.1670\times10^{-5}T^3)\frac{\partial s}{\partial x} \quad (38\text{-s-b})$$

And:

$$\frac{\partial}{\partial x}\left(Gs^{3}/_{2}\right)=(7.944\times10^{-2}+1.6483\times10^{-2}T-5.3009\times10^{-4}T^2)\frac{3}{2}s^{1}/_{2}\frac{\partial s}{\partial x} \quad (40\text{-s-b})$$

Also:

$$\frac{\partial}{\partial x}(Isp)=(2.2838\times10^{-3}-1.0981\times10^{-5}T-1.6078\times10^{-6}T^2)\frac{\partial s}{\partial x}p \quad (44\text{-s-b-})$$

In addition:

$$\frac{\partial}{\partial x}\left(Js^{3}/_{2}p\right)=2.866125\times10^{-4}s^{1}/_{2}\frac{\partial s}{\partial x}p \quad (45\text{-s-b})$$

And:

$$\frac{\partial}{\partial x}(Nsp^2)=(-9.9348\times10^{-7}+2.0816\times10^{-8}T+9.1697\times10^{-10}T^2)\frac{\partial s}{\partial x}p^2 \quad (49\text{-s-b})$$

According to equations (38-s-b). (40-s-b). (44-s-b). (45-s-b) and (49-s-b) by having in mind conditions (53-2) and (53-3):

$$\frac{\partial}{\partial x}[K_T(s.T.p)]=(38\text{-s-b})+(40\text{-s-b})+(44\text{-s-b})+(45\text{-s-b})+(49\text{-s-b}). \quad (50\text{-s-b})$$

Considering conditions (53-2) and (53-3) and substituting (50-s-b) into equation (24-b) yields:

$$\frac{\partial}{\partial x}\left[1-{}^{p}/_{K_T(s.T.p)}\right]=\frac{p(50-\mathrm{s}-\mathrm{b})}{[K_T(s.T.p)]^2} \quad (51\text{-s-b})$$

Substituting (34-s-b) and (51-s-b) in (23-b) yields northward component of the first version of saline current. i.e.:

$$v_{S_{c_I}} = -\frac{g}{f}\int_{z_1}^{z_2} \frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}} \times \frac{(34-s-b)\left[1-{}^{p}/_{K_T(s.T.p)}\right]-\rho(s.T.0)(51-s-b)}{\left[1-{}^{p}/_{K_T(s.T.p)}\right]^2} dz \quad \text{(5-Sc-I-b)}$$

where in equation (5-Sc-I-b). $v_{S_{c_I}}$ is northward component of the first version of saline current. g is acceleration due to gravity. f is Coriolis parameter. z_1 is lower level of the oceanic layer. z_2 is higher level of the oceanic layer. $\rho(s.T.0)$ is density of surface ocean defined in (8). p is pressure of seawater at any point considered. $K_T(s.T.p)$ is mean bulk modulus defined in (9). (34-s-b) is equation with the same number and (51-s-b) is also equation with the same number and z stands for vertical axis of Cartesian coordinates system.

Also. considering conditions (53-2) and (53-3). then combining equations (22-a) and (22-b) yields:

$$\mathbb{v}_{S_{c_I}} = -\frac{g}{f}\int_{z_1}^{z_2} \frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}} \mathbb{k} \times \nabla\left[\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}\right] dz \quad \text{(4-Sc-I)}$$

Equation (4-Sc-I) is <u>*The first version of saline current vector*</u> where in it. $\mathbb{v}_{S_{c_I}}$ is the first version of saline current vector. g is acceleration due to gravity. f is Coriolis parameter. z_1 is depth of lower level of the oceanic layer. z_2 is upper level of the oceanic layer. $\rho(s.T.0)$ is density of surface ocean defined in (8). p is pressure of seawater at any point considered. $K_T(s.T.p)$ is mean bulk modulus defined in (9). $\mathbb{k}$ is vertical unit vector in Cartesian coordinates system. ∇ is gradient operator in Cartesian coordinates system and z is vertical axis in Cartesian coordinates system.

Eastward component of the first version of saline current vector is introduced in equation (5-Sc-I-a) and northward component of it. is presented in equation (5-Sc-I-b). Furthermore. in equation (5-Sc-I-b). derivative of seawater density with respect to x and in equation (5-Sc-I-a) derivative of seawater density with respect to y have done considering conditions (53-2) and (53-3) [by attention to note No. 4]

From the first version of saline current vector or its components. one can see:

2 – 2 – 1 – A: Whatever we go from pole to equator. the first version of saline current becomes stronger (with pay attention to footnote No. 2)

2 – 2 – 1 – B: If the oceanic layer has more thickness; then the first version of the saline current becomes more powerful;

2 – 2 – 1 – C: Whatever oceanic layer is near to ocean surface; density would be lower and the first version of the saline current becomes stronger as a result;

2 – 2 – 1 – D: if the salinity of the oceanic layer would be less – due to decreasing seawater density – the first version of saline current is more powerful and

2 – 2 – 1 – E: If the horizontal gradient of seawater salinity would be greater. the first version of saline current becomes more powerful. because the first version of the saline current is proportional to the horizontal gradient of implicit seawater salinity.

In this manner. the first version of saline current vector. i.e.. equation (4-Sc-I) shows that: "*The first version of saline current flows parallels to Isopycnals and Isopleths of implicit seawater salinity so that. low values of density (low values of salinity) are located at the right side of downstream.*" (In the northern hemisphere) This fact is illustrated in figures 9 and 10.

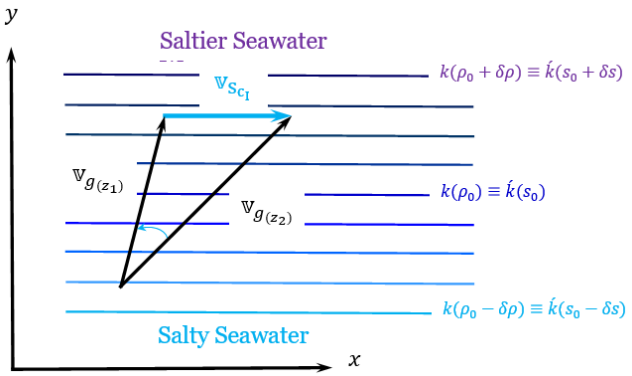

Figure 9. Counterclockwise rotation of geostrophic current with respect to depth (Backing) and less saltiness seawater advection. Coefficient $k = -\frac{g}{f}\langle\frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}}\rangle(z_2 - z_1)$ and $\acute{k}$ is proportional to k in such a manner that – for instance – $k(\rho_0 - \delta\rho)$ is identical to $\acute{k}(s_0 - \delta s)$. [The first version of saline current]

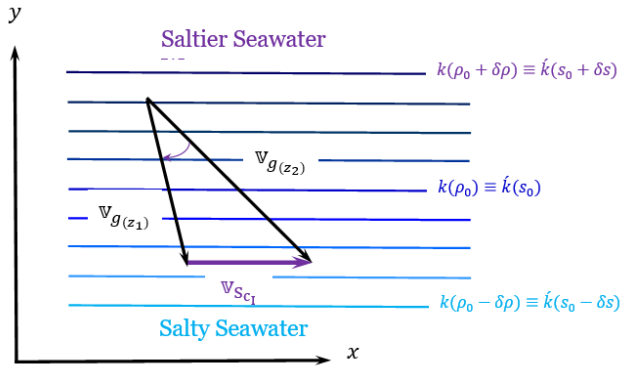

Figure 10. Clockwise rotation of geostrophic current with respect to depth (Veering) and saltier seawater advection. Coefficient $k = -\frac{g}{f}\langle\frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}}\rangle(z_2 - z_1)$ and $\acute{k}$ is proportional to k in such a manner that – for instance – $k(\rho_0 + \delta\rho)$ is identical to $\acute{k}(s_0 + \delta s)$. [The first version of saline current]

In connection with figure 9. counterclockwise turning of geostrophic current with respect to depth (backing) is associated with less saltier seawater advection by geostrophic current in the layer.

Conversely; as shown in figure 10. clockwise turning of geostrophic current with respect to depth (veering) implies advection of saltier seawater by geostrophic

current in the oceanic layer. Also $\langle \frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}} \rangle$ under explanations of figures 9 and 10 is vertical average of specific volume of seawater in the oceanic layer.

2.2.2 The second version of saline current

Regarding to conditions (53-2) and (53-3). we had:

$$u_{S_{c_I}} = \frac{g}{f}\int_{z_1}^{z_2} \frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}} \times \frac{(34-\text{s}-\text{a})\left[1-{}^{p}/_{K_T(s.T.p)}\right]-\rho(s.T.0)(51-\text{s}-\text{a})}{\left[1-{}^{p}/_{K_T(s.T.p)}\right]^2} dz \quad \text{(5-Sc-I-a)}$$

Equation (5-Dc-I-a) was eastward component of the first version of saline current.

In almost all meteorological purposes. we regard acceleration due to gravity as constant. Consequently. we can enter it inside integral sign. considering conditions (53-2) and (53-3); to get eastward component of the second version of saline current i.e.:

$$u_{S_{c_{II}}} = \frac{1}{f}\int_{z_1}^{z_2} \frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}} \times \frac{(34-\text{s}-\text{a})\left[1-{}^{p}/_{K_T(s.T.p)}\right]-\rho(s.T.0)(51-\text{s}-\text{a})}{\left[1-{}^{p}/_{K_T(s.T.p)}\right]^2} d\Phi_{\text{s}} \quad \text{(5-Sc-II-a)}$$

where in equation (5-Sc-II-a). $u_{S_{c_{II}}}$ is eastward component of the second version of saline current. f is Coriolis parameter. $z_1$is lower level of the oceanic layer. z_2 is higher level of the oceanic layer. $\rho(s.T.0)$ is density of surface ocean defined in (8). p is pressure of seawater at any point considered. $K_T(s.T.p)$ is mean bulk modulus defined in (9). (34-s-a) is equation with the same number and (51-s-a) is also equation with the same number. Φ is geopotential and subscript "s" refers to this fact that variation of seawater geopotential is affiliated to seawater salinity solely. with due attention to: [3]

$$\Phi = \Phi(s.T.p) \quad (35)$$

And for deriving formula for the northward component of the second version of saline current. by considering conditions (53-2) and (53-3); we fix the mind on equation (5-Sc-I-b) that was northward component of the first version of saline current i.e.:

$$v_{S_{c_I}} = -\frac{g}{f}\int_{z_1}^{z_2} \frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}} \times \frac{(34-\text{s}-\text{b})\left[1-{}^{p}/_{K_T(s.T.p)}\right]-\rho(s.T.0)(51-\text{s}-\text{b})}{\left[1-{}^{p}/_{K_T(s.T.p)}\right]^2} dz \quad \text{(5-Sc-I-b)}$$

Equation (5-Sc-I-b) was northward component of the first version of saline current. and we can enter acceleration due to gravity inside integral sign to get northward component of the second version of saline current. i.e.:

$$v_{S_{c_{II}}} = -\frac{1}{f}\int_{z_1}^{z_2} \frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}} \times \frac{(34-\text{s}-\text{b})\left[1-{}^{p}/_{K_T(s.T.p)}\right]-\rho(s.T.0)(51-\text{s}-\text{b})}{\left[1-{}^{p}/_{K_T(s.T.p)}\right]^2} d\Phi_{\text{s}} \quad \text{(5-Sc-II-b)}$$

where in equation (5-Sc-II-b). $v_{S_{c_{II}}}$ is northward component of the second version of saline current. f is Coriolis parameter. $z_1$is lower level of the oceanic layer. z_2 is higher level of the oceanic layer. $\rho(s.T.0)$ is density of surface ocean defined in (8). p is pressure of seawater at any point considered. $K_T(s.T.p)$ is mean bulk modulus defined in (9). (34-s-b) is equation with the same number and (51-s-b) is also equation with the same number. Φ is geopotential and subscript "s" refers to this fact that variation of seawater geopotential is affiliated to seawater salinity solely. with due attention to (35)

The first version of saline current vector was:

$$\mathbb{v}_{S_{c_I}} = -\frac{g}{f}\int_{z_1}^{z_2} \frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}} \mathbb{k} \times \nabla\left[\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}\right] dz \quad \text{(4-Sc-I)}$$

If we enter acceleration due to gravity inside integral sign. we get the second version of saline current: i.e.:

$$\mathbb{v}_{S_{c_{II}}} = -\frac{1}{f}\int_{z_1}^{z_2} \frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}} \mathbb{k} \times \nabla\left[\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}\right] d\Phi_{\text{s}} \quad \text{(4-Sc-II)}$$

Equation (4-Sc-II) is <u>*The second version of saline current vector*</u> where in it. $\mathbb{v}_{S_{c_{II}}}$ is the second version of saline current vector. f is Coriolis parameter. z_1 is depth of lower level of the oceanic layer. z_2 is upper level of the oceanic layer. $\rho(s.T.0)$ is density of surface ocean defined in (8). p is pressure of seawater at any point considered. $K_T(s.T.p)$ is mean bulk modulus defined in (9). $\mathbb{k}$ is vertical unit vector in Cartesian coordinates system. ∇ is gradient operator in Cartesian coordinates system. Φ is geopotential and subscript " s " refers to this fact that variation of seawater geopotential is affiliated to seawater saline solely.

Eastward component of the second version of saline current vector is introduced in equation (5-Sc-II-a) and northward component of it. is presented in equation (5-Sc-II-b).

From the second version of saline current vector or its components. one can perceive:

2 – 2 – 2 – A: Whatever we go from pole to equator. the second version of saline current becomes stronger (with pay attention to footnote No. 2)

2 – 2 – 2 – B: If the oceanic layer has more geopotential thickness; then the second version of the saline current becomes more powerful;
2 – 2 – 2 – C: Whatever oceanic layer is near to ocean surface; density would be lower and the second version of the saline current becomes stronger as a result;
2 – 2 – 2 – D: if the salinity of the oceanic layer would be high. the second version of saline current is less powerful and
2 – 2 – 2 – E: If the horizontal gradient of seawater salinity would be greater. the second version of saline current becomes more powerful. because the second version of saline current is proportional to the horizontal gradient of seawater geopotential thickness.
In this manner. the second version of saline current vector. i.e.. equation (4-Sc-II) shows that: "*The second version of saline current flows parallel to Isopycnals and Isopleths of implicit seawater salinity so that. low values of density (low values of salinity and high values of geopotential) are located at the right side of downstream.*" (In the northern hemisphere) This fact is demonstrated in figures 11 and 12.

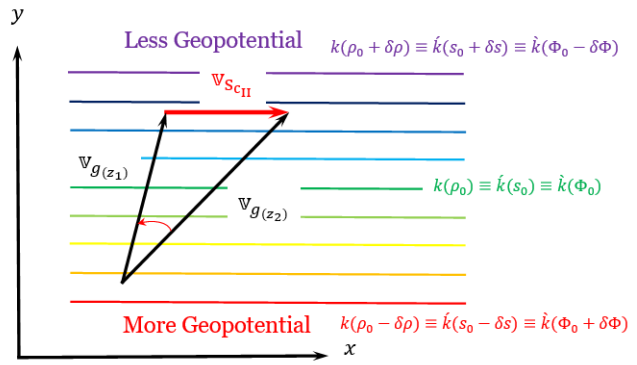

Figure 11. Counterclockwise rotation of geostrophic current with respect to depth (Backing) and more geopotential advection. Coefficient $k=-\frac{1}{f}\langle\frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}}\rangle(\Phi_2-\Phi_1)$. $\acute{k}$ is proportional to k in such a manner that – for instance – $k(\rho_0-\delta\rho)$ is identical to $\acute{k}(s_0-\delta s)$ and $\grave{k}$ is proportional to k in style that – for example – $k(\rho_0-\delta\rho)$ is similar in very detail to $\grave{k}(\Phi_0+\delta\Phi)$ [The second version of saline current]

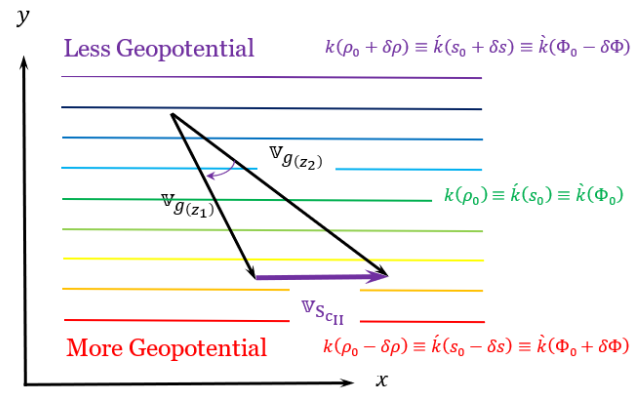

Figure 12. Clockwise rotation of geostrophic current with respect to depth (Veering) and less geopotential advection. Coefficient $k=-\frac{1}{f}\langle\frac{1}{\frac{\rho(s.T.0)}{1-{}^{p}/_{K_T(s.T.p)}}}\rangle(\Phi_2-\Phi_1)$. $\acute{k}$ is proportional to k in such a manner that – for instance – $k(\rho_0+\delta\rho)$ is identical to $\acute{k}(s_0+\delta s)$ and $\grave{k}$ is proportional to k in style that – for example – $k(\rho_0+\delta\rho)$ is similar in very detail to $\grave{k}(\Phi_0-\delta\Phi)$ [The second version of saline current]

In connection with figure 11 – considering conditions (53-2) and (53-3) – counterclockwise turning of geostrophic current with respect to depth (backing) is associated with more geopotential advection in the ocean by geostrophic current in the layer.
Conversely; as shown in figure 12 – considering conditions (53-2) and (53-3) – clockwise turning of geostrophic current with respect to depth (veering) implies less geopotential advection in the ocean by geostrophic current in the oceanic layer.

2.2.3 The third version of saline current

For deriving formulae for the third version of saline current by considering conditions (53-2). (53-3) and this fact that. variation of geopotential is related to variation of seawater salinity only; first we fix the mind on equation (5-Dc-III-a) that was eastward components of the third version of dense current i.e.:

$$u_{D_{c_{III}}}=-\frac{1}{f}\frac{\partial}{\partial y}(\Phi_2-\Phi_1) \qquad \text{(5-Dc-III-a)}$$

In equation (5-Dc-III-a). $u_{D_{c_{III}}}$ is eastward component of the third version of dense current and we used definition (2-Dc-a) for deriving this equation.
And northward component of the third version of dense current was:

$$v_{D_{c_{III}}}=\frac{1}{f}\frac{\partial}{\partial x}(\Phi_2-\Phi_1) \qquad \text{(5-Dc-III-b)}$$

In equation (5-Dc-III-b). $v_{D_{c_{III}}}$ is northward component of the third version of dense current and we used definition (2-Dc-b) for extracting this equation.
With due attention to: [3]

$$\Phi=\Phi(s.T.p) \qquad (35)$$

and emphasis on conditions (53-2) and (53-3) based upon that horizontal variation of seawater density is related to horizontal variation of seawater salinity solely. and implies variation of seawater geopotential height only by horizontal variation of seawater salinity. we get:

$$u_{S_{c_{III}}}=-\frac{1}{f}\frac{\partial}{\partial y}\left(\Phi_{s_2}-\Phi_{s_1}\right) \qquad \text{(5-Sc-III-a)}$$

where. in equation (5-Sc-III-a). $u_{S_{c_{III}}}$ is eastward component of the third version of saline current. f is Coriolis parameter. y is northward axis of Cartesian coordinates system. Φ_{s_2} is geopotential of higher level of the oceanic layer. Φ_{s_1} is geopotential of lower level of the oceanic layer and $\left(\Phi_{s_2}-\Phi_{s_1}\right)$ is geopotential thickness of the oceanic layer. Furthermore. subscript "s" refers to this fact that increasing or decreasing of

geopotential is related on seawater salinity only. with due attention to (35).
and:

$$v_{S_{c_{III}}} = \frac{1}{f}\frac{\partial}{\partial x}\left(\Phi_{s_2} - \Phi_{s_1}\right) \qquad \text{(5-Sc-III-b)}$$

where. in equation (5-Sc-III-b). $v_{S_{c_{III}}}$ is northward component of the third version of saline current. f is Coriolis parameter. x is eastward axis of Cartesian coordinates system. Φ_{s_2} is geopotential of higher level of the oceanic layer. Φ_{s_1} is geopotential of lower level of the oceanic layer and $\left(\Phi_{s_2} - \Phi_{s_1}\right)$ is geopotential thickness of the oceanic layer. Furthermore. subscript "s" refers to this fact that increasing or decreasing of geopotential thickness is related on seawater salinity only. with due attention to (35).
Now; by joining (5-Sc-III-a) and (5-Sc-III-b) as vector. we obtain:

$$\mathbb{v}_{S_{c_{III}}} = f^{-1}\mathbb{k} \times \nabla_p\left(\Phi_{s_2} - \Phi_{s_1}\right) \qquad \text{(4-Sc-III)}$$

where. in equation (4-Sc-III). $\mathbb{v}_{S_{c_{III}}}$ is: *The third version of saline current vector*. f is Coriolis parameter. $\mathbb{k}$ is vertical unit vector in pressure coordinates system. ∇_p is gradient operator in pressure coordinates system. Φ_{s_2} is geopotential of higher level of the oceanic layer. Φ_{s_1} is geopotential of lower level of the oceanic layer and $\left(\Phi_{s_2} - \Phi_{s_1}\right)$ is geopotential thickness of the oceanic layer. Furthermore. subscript "s" refers to this fact that increasing or decreasing of geopotential thickness is related on seawater salinity only. with due attention to (35).
From the third version of saline current vector or its components. one can perceive:
2 – 2 – 3 – A: Whatever we go from pole to equator. the third version of saline current becomes stronger (with pay attention to footnote No. 2)
2 – 2 – 3 – B: If the oceanic layer has more geopotential thickness; then the third version of the saline current becomes more powerful. and
2 – 2 – 3 – C: If the horizontal gradient of seawater geopotential thickness would be greater. the third version of saline current becomes more powerful. because the third version of the saline current is proportional to the horizontal gradient of seawater geopotential thickness.
In this manner. the third version of saline current vector. i.e.. equation (4-Sc-III) shows that: "*The third version of saline current flows parallel to Isopleths of geopotential thickness of the oceanic layer* i.e.. $(\Phi_{s_2} - \Phi_{s_1})$ *so that. high values of geopotential thickness are located at the right side of downstream.*" (In the northern hemisphere) This fact is illustrated in figures 13 and 14.

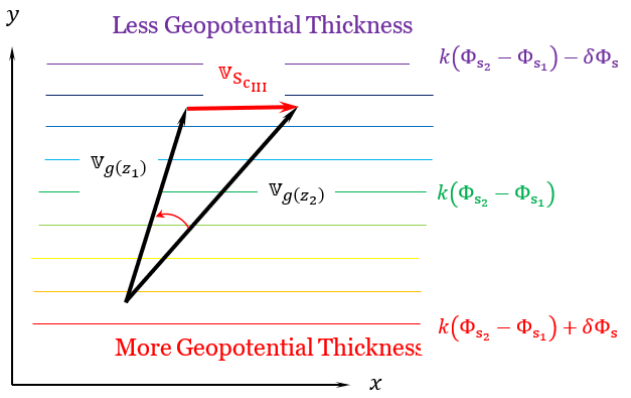

Figure 13. Counterclockwise rotation of geostrophic current with respect to depth (Backing) and more geopotential thickness advection. Coefficient $k = \frac{1}{f}$ [The third version of saline current]

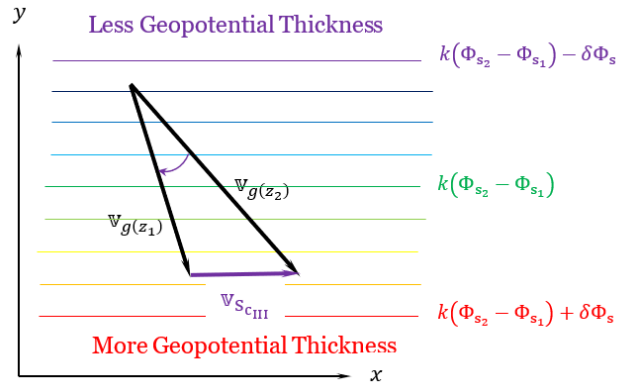

Figure 14. Clockwise rotation of geostrophic current with respect to depth (Veering) and less geopotential thickness advection. Coefficient $k = \frac{1}{f}$ [The third version of saline current]

In connection with figure 13 – considering conditions (53-2) and (53-3) – counterclockwise turning of geostrophic current with respect to depth (backing) is associated with more geopotential thickness advection of the oceanic layer by geostrophic current in the layer. Conversely; as shown in figure 14 – considering conditions (53-2) and (53-3) – clockwise turning of geostrophic current with respect to depth (veering) implies less geopotential thickness advection of oceanic layer by geostrophic current in the oceanic layer.

3. Results and Discussion

All versions of the dense current equations are extremely useful diagnostic tools. which is often used to check analyses of the observed current field for consistency.
It can also be used to estimate the mean horizontal oceanic seawater density. and thickness advections in an oceanic layer.
Dense current is struggle of the oceanic medium to return thermodynamic equilibrium and completing the dynamic cycle of ocean. This movement begins from the fact that solar radiation disturbs the thermodynamic

equilibrium of oceanic ambience. resulting production of horizontal gradient of density. Horizontal gradient of density produces horizontal gradient of potential energy and in turn. this condition forces ocean to generate horizontal gradient of pressure and finally. it causes to start current for returning thermodynamic equilibrium of the ocean.

Concerning various insolation and non-uniform transfer of diffusion of heat and salinity in the different layers of the ocean; horizontal gradients of density are not same in the oceanic layers and current velocities cannot be the same at oceanic layers. Therefore. this phenomenon produces dense current that is effort of ocean to reduce horizontal gradient of density and in turn. reducing dense current speed. By continuous reduction of dense current speed. thermodynamic disequilibrium of ocean weakens and weakens. until returning the thermodynamic equilibrium of the ocean. If we assume there will be no more solar radiation. finally the ocean current will be disappeared in presence of friction.

Some time. horizontal variation of seawater density is a result of horizontal variations of seawater temperature. This phenomenon produces a special case of dense current that we call it *Thermal current.*

Likewise. in some other case; the horizontal variation of seawater density is related to horizontal variation of seawater salinity only. This phenomenon produces another special case of dense current that we call it *Saline current.*

Thermal current and saline current can create under thermocline layer in some special conditions as well as dense current.

So. dense current and its special cases i.e.. thermal current and saline current those produced by baroclinic domain. are main response of the oceanic environment for returning thermodynamic equilibrium again. This means that. oceanic currents including geostrophic currents transfer high density seawater to location of low-density seawater. transfer high seawater temperature to low seawater temperature. likewise high seawater salinity toward seawater with low salinity region and analogically vice versa.

Therefore. transferring high density seawater to location of low-density seawater; moving high temperature seawater to region with low seawater temperature; shift seawater with high salinity to region with low salinity seawater. as well as advecting more thickness oceanic layer toward less thickness oceanic layer; have important roles for diffusing heat and salt in oceans for the purpose of balancing temperature and salinity in oceans. These conditions are important duties of oceans toward regulating of world climate balance.

Also. dense current vector equations. equations of eastward component of dense current and equations of northward component of dense current show:

3 – 1 – At any time density varies in horizontal direction. by horizontal variation of temperature. salinity or pressure. i.e.. all time we confront by baroclinic medium; therefore. always there are dense current. thermal current or saline current too;

3 – 2 – Whatever we approach to tropical latitudes. dense current. thermal current or saline current will be powered (with pay attention to footnote No. 2);

3 – 3 – Whatever we close to ocean surface – for reason of light density - dense current. thermal current or saline current has more force;

3 – 4 – If there is high horizontal gradient of density. high horizontal gradient of temperature or more horizontal gradient of salinity; dense current. thermal current or saline current has more strength respectively. and

3 – 5 – If there is more horizontal gradient of thickness of the oceanic layer; then dense current. thermal current or saline current has more power.

Likewise. the propositions of dense current. thermal current or saline current those are propounded with above-mentioned configurations; can help to better understanding of ocean dynamics. Furthermore. knowing the advections of the heavy seawater density. light seawater density. high temperature seawater. low temperature seawater. high salinity seawater. low salinity seawater or advection of the thickness of the oceanic layer with exclusive specifications; can help to improve oceanic forecasting. Although. we referred to the variation of the geostrophic current with respect to depth here; but this proposition is valid for the variation of real oceanic current in vertical direction with some modifications that needs separate discussion.

And study of dense current. thermal current or saline current can enlighten deep sea dynamics and helps to better understanding climate of deep oceans.

4. Conclusions

Ocean currents are powerful and dynamic systems that drive the Earth’s climate. affect marine life. and transfer vast amounts of energy around the globe. A better understanding of these currents can greatly improve our ability to predict and respond to extreme weather events while helping us make smarter decisions when planning for future growth and development. It is also important to remember that ocean currents play a key role in regulating global temperatures by moving heat from the equator to poleward regions as well as regulating global ocean’s salinity.

Many phenomena and mechanisms have essential roles for transfer and diffusion of arriving solar radiation from tropical regions to high latitudes in northern hemisphere and low latitudes in southern hemisphere.

The most important of these mechanisms are: winds. ocean currents. sensible heat. latent heat. monsoon phenomenon (as other type of latent heat transfer).

tropical cyclones. meridional overturning circulation. Rossby waves and Antarctic circumpolar current.

Oceanic currents are second mechanism for transferring sensible heat of solar radiation from tropical regions toward high or low latitudes of the earth in northern hemisphere and southern hemisphere. And; in comparison with the first mechanism for transferring sensible heat from tropical regions to high or low latitudes. i.e.. wind; the minimum transferring of heat by ocean current per unit volume is 3395 times and maximum transferring heat by ocean current per unit volume is 4688 times as long as transferring heat by wind.

Therefore. research about dense current. thermal current or saline current is vital for understanding heat and saltiness balance in oceans and study of world climate via deep oceans.

In classification; dense current. thermal current and saline current are classified in thermohaline circulation. This kind of ocean circulation is driven by differences in water density. The cold. dense water sinks to the ocean floor and is replaced by warm. less dense water from the surface. This movement of water can occur on a global scale. with water sinking in the polar regions and rising in the tropics. as well as on a more local scale.

Overall. these currents play a vital role in the ocean's ecosystem. climate and weather patterns. and human activities. They also have a major influence on the distribution of heat. carbon and other elements in the ocean. and they support marine life. Understanding these currents is important for managing resources and predicting future changes in the ocean's environment.

Atmospheric movements (winds) and oceanic movements (currents) are some parts of the mechanisms those try to adjust heat. humidity and salinity in Atmosphere-Ocean System. Even different shear of them (movements) including their vertical shear; have basic roles in the subject.

Our goals in this article were familiarizing with oceanic efforts for transfer of sensible heat via dense current. thermal current and saline current those are vertical shear of geostrophic current in baroclinic ocean with specific conditions. And this work helps us to better realize oceanic climate.

Separation of oceanic currents same as real oceanic current. geostrophic current. dense current. thermal current and saline current is not easy. To separate them for understanding their effects. we need to use oceanic model with high accuracy for knowing the type of ocean current. And preparing this kind oceanic model. is essential necessity and vital for our society.

It is necessary to note the basic point. Atmosphere and oceanic waters are baroclinic. And the theory of barotropic ambience – same as geostrophic balance – is acceptable for simplification of meteorological and oceanic analyses.

However. baroclinic environment. geostrophic current and variation of it with respect to depth. definition of thermal current and saline current. deriving of thermal current vector in Cartesian coordinates system as well as deriving saline current vector in this coordinates system. the other view to derive equation of thermal current and saline current. other view of relation between thermal current and geopotential thickness. also relation between saline current and geopotential thickness. have been discussed expanded upon the article.

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