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A New Look at the Vertical Shear of the Geostrophic Current Part I: Dense Current

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ABSTRACT

Many appropriate and necessary phenomena and mechanisms have essential roles for transfer and diffusion of arriving solar radiation, from tropical regions to high latitudes in northern hemisphere and low latitudes in southern hemisphere.
Atmospheric movements (winds) and oceanic movements (currents) are some parts of these mechanisms. Even different shear of them (movements) including vertical shear of them; have basic roles in the subject.
Our goal in this research is to familiarize with oceanic efforts for transfer of sensible heat via dense current that is vertical shear of geostrophic current.
In connection with the subject; three versions of dense current were introduced. Furthermore; baroclinic environment, geostrophic current and variation of it with respect to depth, definition of dense current, deriving of dense current vector in Cartesian Coordinates System, the other view to derive other equations of dense current, relation between dense current and geopotential thickness, other view of relation between dense current and geopotential thickness, and other subject related to dense current; have discussed expanded upon the research. Also, study of dense current can enlighten deep sea dynamics and helps to better understanding the climate of deep oceans.

1. Introduction

The majority of incoming solar radiation enter at tropical regions of the earth. Absorbed solar heat in tropical region, transfers or diffuses via some mechanisms, to high latitudes in northern hemisphere and low latitudes in southern hemisphere. The most important of these mechanisms are: winds, ocean currents, sensible heat, latent heat, monsoon phenomenon (as other type of latent heat transfer), tropical cyclones, meridional overturning circulation, Rossby waves and Antarctic circumpolar current.
These essential mechanisms; are agents of produce other sub-mechanisms or phenomena themselves too; those have a special role for heat transfer or heat diffusion directly or indirectly.
Especially, all atmospheric or oceanic movements, have a specific role or diverse roles for transferring heat from tropical regions to other parts of the earth. Even, variation of these movements, so that, assessed in this manner. For instance, variations of wind in vertical direction and variations of oceanic current with respect to depth, are inside of these struggles those arrived heat in the earth, doesn't remain in one region and transferred or diffused to another regions in order to earth will be usable for living human beings and other living beings.
Although variation of wind with respect to height investigated many times and, in this field, observational studies as well as researches have been done since the beginning of the last century; variation of oceanic currents with respect to depth have been rarely investigated. For instance, recently, Zamanian introduced dense wind [1] and, thermal wind and moist wind [2] as results of vertical shear of geostrophic wind and showed how light air, dense air, heat or moisture advects in atmosphere to discover response time for returning of the; thermodynamic equilibrium.
Now, it is necessary to do observational studies and researches in this important field, i.e., variations of oceanic currents with respect to depth. There are various profits for research on variation of the oceanic current with respect to depth in oceans. In the first place, it is used for detection of advection of light or

heavy seawater; and in the second stage, it helps to verify current observation and in the third place, detecting oceanic phenomenon type, because vertical shear of oceanic current will happen in condition that ocean water is baroclinic fluid and in this circumstance; we can track returning the oceanic thermodynamic equilibrium. Therefore, there is an urgent need for careful studies of variations of oceanic current with respect to depth, to decipher many oceanic phenomena. For reminding, baroclinic oceanic environment is an oceanic water that density varies in horizontal direction [1] and is not motionless, because in this medium; the lack of thermodynamic equilibrium exists and for this circumstance, fluid cannot be stationary. In this kind basin, current varies in vertical direction. These variations and disequilibrium are main factors for returning thermodynamic equilibrium to the oceanic environment again. In addition, horizontal gradient of density directly contributes to the oceanic thermohaline circulation.

Sometimes investigation of variations of current with respect to depth is a tool for considering heat transfer or salinity transfer, and sometimes is a means for research on momentum transfer. Undoubtedly, because atmospheric system or oceanic system are systems that all their parameters related to each other, and in fact, both with together is one system that we identify it as Atmospheric-Oceanic System (AOS); therefore, study of variations of current with respect to depth can possess many other applications in oceanic basin, as well as atmospheric environment.

In connection with the subject as a matter of fact; although the main axis of this study should be variations of oceanic current with respect to depth; anyhow, for simplification of subject, we can derive a benefit from geostrophic current for explaining these transformations.

The main objectives of this research are:

To know what is baroclinic ocean;

To recognize reasons for variation of geostrophic current in vertical coordinate;

To introduce dense current in oceans;

To know dense current equations;

To understand what is the role of dense current in oceans? and

Introducing some oceans' struggle for transfer of sensible heat energy from a region to another region via dense current

Ocean water moves in two directions: horizontally and vertically. Horizontal movements are referred to as currents, while vertical changes in position are called upwelling or downwelling.

The ocean currents may be classified based on their depth; as surface currents and deep-water currents. Surface currents constitute about 10 percent of all the water in the oceans, these waters are the upper 400 m of the oceans; whereas deep water currents make up the other 90 percent of the oceans' water. These waters move around the ocean basins due to variations in density and gravity, in the manner that density difference is a function of different temperature or salinity

These deep waters sink into the deep ocean basins at high latitudes where the temperatures are low enough to cause the density to increase.

The ocean depths are crucial to the world's climate and ecosystems. They act as 'carbon sinks', store heat, and transport things like oxygen and nutrients around the globe via currents. However, only 10 per cent of ocean data comes from below 2000m, so we know very little about how significant the role of the deep ocean is at regulating the climate system.

For instance, New Zealand's environment is greatly impacted by deep-ocean processes, with warm subtropical currents coming from Australia and the cold Antarctic circumpolar current coming in from the south. Therefore, by understanding the deep ocean, we understand our entire climate and environment better. [3]

2. Preliminary beginning

Ocean currents are the continuous, predictable, directional movement of seawater. It is a massive movement of ocean water that is caused and influenced by various forces. They are like river flows in oceans.

Primary forces for oceanic currents are wind, tide, outflow of rivers and changing density mainly due to solar radiation.

It is convenient to split the oceanic velocity field into two parts. The first is the *wind-driven* part, $\mathbb{u}_E$ defined by the relation:

$$\rho_0 f \mathbb{k} \times \mathbb{u}_E = \frac{\partial \boldsymbol{\tau}}{\partial z} \tag{1}$$

where in equation (1), ρ_0 is average value of density, f is Coriolis parameter, $\mathbb{k}$ is vertical unit vector in Cartesian coordinates system, $\mathbb{u}_E$ is wind-driven part of velocity, subscript E refers to Ekman, $\boldsymbol{\tau}$ is turbulent stress vector in the mixed layer and z is vertical axis of Cartesian coordinates system.

We assume that $\boldsymbol{\tau}$ and thus also $\mathbb{u}_E$ vanish below the depth of the mixed layer d_m.

The *geostrophic* part of the velocity is similarly defined as $\mathbb{u}_G$ and satisfies:

$$\rho_0 f \mathbb{k} \times \mathbb{u}_G = -\nabla p \tag{2}$$

where in equation (2) p is pressure, so that [4]:

$$\mathbb{u} = \mathbb{u}_G + \mathbb{u}_E \tag{3}$$

[1] *Horizontal direction or horizontal surface is the surface parallel to Mean Sea Level (M.S.L.)*

where in equation (3) $\mathbb{u}$ is total oceanic current.

3. Geostrophic Current

In Cartesian coordinates system, geostrophic current can be introduced by: [5]

$$\mathbb{v}_g \equiv \mathbb{k} \times \frac{1}{\rho f} \nabla p \tag{4}$$

where in equation (4) $\mathbb{v}_g$ is geostrophic current vector, $\mathbb{k}$ is upward unit vector in Cartesian coordinates system, ρ is density of seawater, f is Coriolis parameter and p stands for pressure.
If we derive eastward and northward components of geostrophic current, we get:

$$u_g = -\frac{1}{\rho f}\frac{\partial p}{\partial y} \tag{5-a}$$

and

$$v_g = \frac{1}{\rho f}\frac{\partial p}{\partial x} \tag{5-b}$$

where in equation (5-a) u_g is eastward component of geostrophic current and in equation (5-b) v_g is northward component of geostrophic current.
Equation (4) shows that geostrophic current vector is proportional to pressure gradient and parallel to isobars, so that high pressure is located on the right side of downstream and low pressure is situated on the left side of downstream (in the north hemisphere) and the proportional coefficient is $\frac{1}{\rho f}$. Figure 1 shows geostrophic current in Cartesian coordinates system.

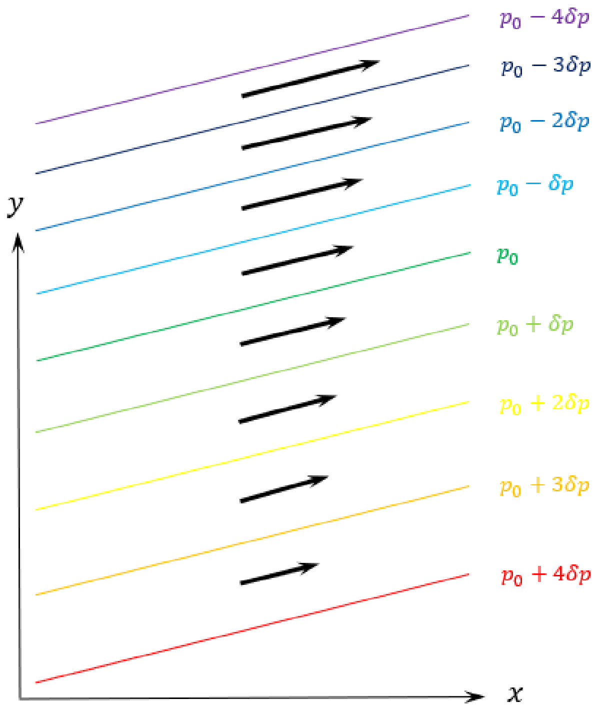

Figure 1. Geostrophic current in Cartesian coordinates system; parallel to isobars.

In practice; most of researchers use below equation as hydrostatic equation in Cartesian coordinates system:

$$\frac{dp}{dz} = -\rho g \tag{6}$$

where in equation (6) p is pressure, z is vertical axis of Cartesian coordinates system, ρ is density and g is acceleration due to gravity. Multiplying both sides of equation (6) by dz we get:

$$dp = -\rho g dz \tag{7}$$

We fix the mind on a layer of oceanic water that its lower level has p_1 pressure and $z_1$height; and its upper level has p_2 pressure and z_2 height as depicted in figure 2.

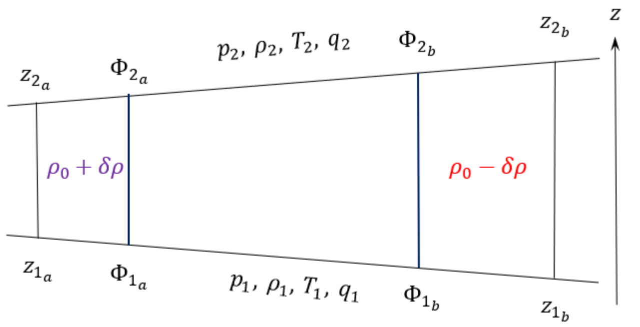

Figure 2. Typical cross section of atmospheric or oceanic layer. [2]

By integrating equation (7) from lower level to upper level of the layer, we get:

$$(p_1 - p_2) = \bar{\rho} g (z_2 - z_1) \tag{8}$$

where in equation (8) $\bar{\rho}$ is average density of the layer and by means of neglecting variation of acceleration due to gravity in vertical direction for meteorological purposes. Noting that $p_2 < p_1$ and $z_1 < z_2$; if we call $(p_1 - p_2) = \delta p$, the partial pressure of column of air in the layer; and $(z_2 - z_1) = \delta z$, the partial height or thickness of the layer; from equation (8) and these assumptions, we have:

$$\delta p = \bar{\rho} g \delta z \tag{9}$$

Looking to equation (9) shows height is function of pressure, density and acceleration due to gravity, same below:

$$\delta z = {}^{\delta p}/_{\bar{\rho} g} \tag{10}$$

By means of equation (10); whenever we pass on isobaric surface and taking in account constant acceleration due to gravity; increasing $\bar{\rho}$ decreases δz and decreasing $\bar{\rho}$ increases δz. [1] Therefore, for the reason of variation of temperature's horizontal gradient or salinity's horizontal gradient on isobaric surface; there is the horizontal gradient of height, because according to the equation of state for seawater – that will introduce afterwards – density of seawater, according to the equation of state, is a function of pressure, temperature and salinity. [6] and [7] i.e.:

$$\rho = \rho(s.T.p) \tag{11}$$

In oceanic circumstance, variation of geostrophic current with respect to depth will be occurred, whenever, the field would be baroclinic ambience and it is clear that the reason of baroclinity is "*the existence of horizontal gradient of density*". For oceans in a definite depth; the horizontal gradient of density is related to the horizontal gradient of temperature as well as the horizontal gradient of salinity or in general; both of them. In oceans – especially – under pycnocline layer – that can be extended until thousand meters below the oceans' surface – if there is the horizontal gradient of density because of existence of horizontal gradient of temperature, horizontal gradient of salinity or both of them; so, the field is baroclinic. Therefore; it is necessary that variation of geostrophic current with respect to depth would be study carefully. Figure 3 shows density structure of the oceans and profile of vertical direction of ocean.

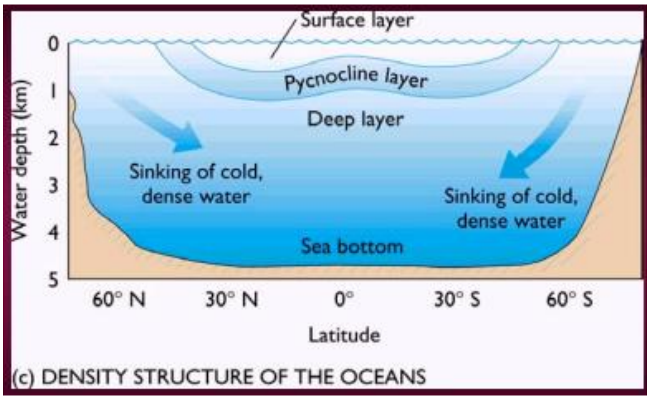

Figure 3. Density structure of the oceans [8]

If in vertical direction; horizontal gradient of pressure, for the reason that existence of horizontal gradient of density and this case for the reason that existence of horizontal gradient of temperature or existence of horizontal gradient of salinity varies; then geostrophic current changes in vertical direction likewise, because oceanic water has become baroclinic. Furthermore, regarding to equation (5-b); if we want northward component of geostrophic current magnitude becomes greater with respect to height, requires isobaric surface gradient increase with increasing height in x direction. According to equation (10), the thickness of the layer between two isobaric surfaces is proportional to the density in the layer. In figure 4 the mean density ρ_1 of the column denoted by δz_1 must be greater than the mean density ρ_2 for the column denoted by δz_2. Hence, an increase with height of a positive *x* directed pressure gradient, must be associated with a negative *x* directed density gradient. The seawater in a vertical column at x_2, because it has less dense (warmer or less salinity), must occupy a greater depth for a given pressure drop than the seawater at x_1. Depiction of this fact can see in figure 4.

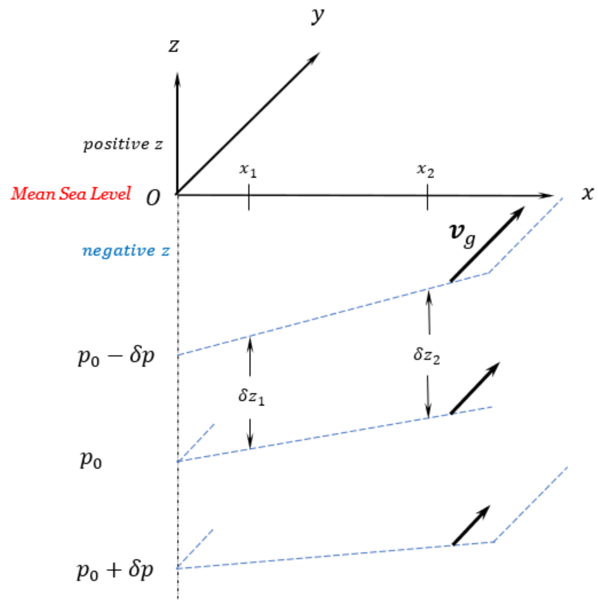

Figure 4. Relationship between vertical shear of the geostrophic current and horizontal thickness gradients.

4. Dense current

The characteristic of baroclinic field, is that current changes with respect to depth, because of existence of the horizontal gradient of density. In this field, the geostrophic current varies with respect to depth as well, as we can see in figure 4.

Existence of horizontal gradient of temperature or presence of horizontal gradient of salinity in oceanic waters – as a general case – implies horizontal gradient of density and the result is performing ocean as baroclinic domain. If for the reason that executing of ocean baroclinity is the existence of horizontal gradient of temperature or the presence of horizontal gradient of salinity; geostrophic current varies with respect to depth and dense current is its result.

4.1. Pre-Assumptions

We need to express pre-assumptions for dense current and focus on movement that obtains power from the horizontal gradient of density solely. In this manner; we should exclude wind driven current, Ekman spiral, tidal current and river outflows. We emphasis this current should start only by baroclinic ocean not other forces. Therefore; one can categorize dense current as deep current of ocean frequently.

4.2. Definition

"*Dense current is vectorial difference of geostrophic current vector at upper level and geostrophic current vector at lower level*" (of the oceanic layer), [see figure 2] providing no more forces to produce oceanic current same as wind, tidal force, or river outflow exception horizontal gradient of density; that is:

$$\mathbb{V}_{\mathrm{D_c}} \equiv \mathbb{V}_{g(z_{2_a})} - \mathbb{V}_{g(z_{1_a})} \tag{12}$$

where in equation (12) $\mathbb{v}_{\mathrm{D_c}}$ stands for dense current vector, $\mathbb{v}_{g(z_{2_a})}$ is geostrophic current at upper level (at less depth), and $\mathbb{v}_{g(z_{1_a})}$ is geostrophic current at lower level (at more depth) where z_{2_a}refers to level with less depth and z_{1_a} refers to level with more depth. [see figure 2]

According to definition (12); eastward and northward components of dense current can be shown as following:

$$u_{\mathrm{D_c}} = u_{g(z_{2_a})} - u_{g(z_{1_a})} \tag{13-a}$$

and

$$v_{\mathrm{D_c}} = v_{g(z_{2_a})} - v_{g(z_{1_a})} \tag{13-b}$$

where in equation (13-a), $u_{\mathrm{D_c}}$ is eastward component of dense current, $u_{g(z_{2_a})}$ is eastward component of geostrophic current in less depth and $u_{g(z_{1_a})}$ is eastward component of geostrophic current in more depth.

In this manner, in equation (13-b), $v_{\mathrm{D_c}}$ is northward component of dense current, $v_{g(z_{2_a})}$ is northward component of geostrophic current in less depth and $v_{g(z_{1_a})}$ is northward component of geostrophic current in more depth.

Despite its name, dense current, while a vector, is not a true current. Instead, it is a geostrophic current shear, representing the change of geostrophic current with respect to depth (or height), causing some advections.

In the condition that wind blows on ocean surface; if environment is baroclinic, geostrophic current shear can obtain power from Ekman spiral as well as baroclinity of domain, and variation of geostrophic current with respect to depth is resultant of both forces. In this condition, force of Ekman spiral – especially in less depth – has more power naturally; and furthermore, transfer of momentum from atmosphere to ocean frustrates geostrophic theory. However, in this work we assume wind doesn't blow on ocean surface and variations of geostrophic current with respect to depth, obtains power from baroclinity of domain solely. Particularly, this subject is true for neighborhood of 30 degrees latitudes both in northern hemisphere and southern hemisphere, when there is no atmospheric system and wind doesn't blow. Of course; observations should confirm this phenomenon.

Ekman spiral – dense current interaction is more precise discussion that is not the case for this work, and now we proceed to explain dense current only; on the conditions those are:

4 – 2 – 1 – Assuming that wind doesn't blow over ocean;

4 – 2 – 2 – Assuming that no more tidal force i.e., in amphidromic region;

4 – 2 – 3 – Assuming that no more river's outflow;

4 – 2 – 4 – Assuming that no more other current in ocean's depths and

4 – 2 – 5 – Assuming the current is geostrophic and variation of geostrophic current with respect to depth enforces form baroclinic ocean only. (This case is true in the neighborhoods of latitudes -30 and +30. (But it should examine by observations.)

The above-mentioned assumptions are basics and true in the complete of this article.

4.3. Formulation of dense current in Cartesian coordinates system

The equations of eastward and northward components of geostrophic current were:

$$u_g = -\frac{1}{\rho f}\frac{\partial p}{\partial y} \tag{5-a}$$

and

$$v_g = \frac{1}{\rho f}\frac{\partial p}{\partial x} \tag{5-b}$$

If we differentiate equation (5-a) in vertical direction, we get:

$$\frac{\partial u_g}{\partial z} = -\frac{1}{f}\left[\frac{\partial}{\partial z}\left(\frac{1}{\rho}\frac{\partial p}{\partial y}\right)\right] = -\frac{1}{f}\left[\frac{1}{\rho^2}\left(-\frac{\partial \rho}{\partial z}\right)\frac{\partial p}{\partial y} + \frac{1}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial p}{\partial z}\right)\right] \tag{14}$$

Substituting equivalent of $\frac{\partial p}{\partial z}$ form equation (6) into equation (14) yields:

$$\frac{\partial u_g}{\partial z} = \frac{1}{f}\left[\frac{g}{\rho}\frac{\partial \rho}{\partial y} + \frac{1}{\rho^2}\frac{\partial \rho}{\partial z}\frac{\partial p}{\partial y}\right] \tag{15-a}$$

And with applying same method for equation (5-b) we get:

$$\frac{\partial v_g}{\partial z} = -\frac{1}{f}\left[\frac{g}{\rho}\frac{\partial \rho}{\partial x} + \frac{1}{\rho^2}\frac{\partial \rho}{\partial z}\frac{\partial p}{\partial x}\right] \tag{15-b}$$

Equations (15-a) and (15-b) show that the rate of geostrophic current with respect to depth is forced by sum of two terms. First terms of both equations, are proportional to horizontal gradient of density and in the manner, - and we will show afterwards in introducing baroclinic vector that - second terms of equations (15-a) and (15-b) are representative of baroclinic ambience partly, and for the reason that environment is baroclinic, these terms are produced. In addition, the main reason for existence of these terms is presence of horizontal gradient of density.

Now if we calculate acceleration due to gravity from hydrostatic equation, i.e.,

$$\frac{\partial p}{\partial z} = -\rho g \tag{6}$$

that is:

$$g = -\frac{1}{\rho}\frac{\partial p}{\partial z} \tag{16}$$

and substitute in equations (15-a) and (15-b), we get:

$$\frac{\partial u_g}{\partial z} = \frac{1}{f}\left[\frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial z}\frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y}\frac{\partial p}{\partial z}\right)\right] \quad (17)$$

or:

$$\frac{\partial u_g}{\partial z} = -\frac{1}{f}\left[\frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial y}\frac{\partial p}{\partial z} - \frac{\partial \rho}{\partial z}\frac{\partial p}{\partial y}\right)\right] \quad (18\text{-a})$$

and:

$$\frac{\partial v_g}{\partial z} = -\frac{1}{f}\left[\frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial z}\frac{\partial p}{\partial x} - \frac{\partial \rho}{\partial x}\frac{\partial p}{\partial z}\right)\right] \quad (18\text{-b})$$

With a view to a careful examination the matter on hand and taking care of subject better; we should think about baroclinic vector with precision.
If the horizontal surface and isopycnal surface do not coincide, we name this state of fluid as baroclinic conventionally.
Now we pay attention to baroclinic vector: [9], [10] and [11] that is:

$$\mathbb{b} = \frac{1}{\rho^2}(\nabla\rho \times \nabla p) \quad (19)$$

where in equation (19), $\mathbb{b}$ is baroclinic vector, ρ is density, p is pressure and ∇ is gradient operator. If the fluid is baroclinic, then the baroclinic vector exists and the relative circulation will change with time if the average normal component of this vector on a surface same A, is different from zero. [10]
By writing components of baroclinic vector, we have:

$$\mathbb{b} = \frac{1}{\rho^2}\begin{bmatrix} \mathbb{i}\left(\frac{\partial \rho}{\partial y}\frac{\partial p}{\partial z} - \frac{\partial \rho}{\partial z}\frac{\partial p}{\partial y}\right) + \mathbb{j}\left(\frac{\partial \rho}{\partial z}\frac{\partial p}{\partial x} - \frac{\partial \rho}{\partial x}\frac{\partial p}{\partial z}\right) + \\ \mathbb{k}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right) \end{bmatrix} \quad (20)$$

We see that right side term of equation (18-a) is proportional to the first component of baroclinic vector and right side term of equation (18-b) is proportional to the second component of baroclinic vector and coefficient of proportionality for both of them is “$-\frac{1}{f}$”
Therefore, both right side terms of equations (18-a) and (18-b) have been produced because fluid medium is baroclinic. In other word; the rate of change of geostrophic current’s components in vertical direction gain force from baroclinic domain.
Now, by fixing the mind on equations (18-a) and (18-b); it is clear that variation of u_g with respect to vertical direction is proportional to the first component of baroclinic vector; and variation of v_g with respect to vertical direction is proportional to the second component of baroclinic vector exactly and coefficient of proportionality for both of them is $-\frac{1}{f}$. Furthermore; these two equations show when variations of components of geostrophic current with respect to vertical direction is possible that, fluid environment is baroclinic, because both terms in brackets of equations (18-a) and (18-b) are first and second components of baroclinic vector respectively.
With pay attention to figure 2; if we integrate equations (18-a) and (18-b) with respect to depth, from higher depth z_1 with higher pressure $p_1$to lower depth z_2 with lower pressure p_2; we get:

$$\int_{z_1}^{z_2}\frac{\partial u_g}{\partial z}dz = -\frac{1}{f}\int_{z_1}^{z_2}\left[\frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial y}\frac{\partial p}{\partial z} - \frac{\partial \rho}{\partial z}\frac{\partial p}{\partial y}\right)\right]dz \quad (21\text{-a})$$

and:

$$\int_{z_1}^{z_2}\frac{\partial v_g}{\partial z}dz = -\frac{1}{f}\int_{z_1}^{z_2}\left[\frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial z}\frac{\partial p}{\partial x} - \frac{\partial \rho}{\partial x}\frac{\partial p}{\partial z}\right)\right]dz \quad (21\text{-b})$$

or:

$$u_{g_{z_2}} - u_{g_{z_1}} \equiv u_{\mathrm{D_{c_I}}} = -\frac{1}{f}\int_{z_1}^{z_2}\left[\frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial y}\frac{\partial p}{\partial z} - \frac{\partial \rho}{\partial z}\frac{\partial p}{\partial y}\right)\right]dz \quad (22\text{-Dc-I-a})$$

and:

$$v_{g_{z_2}} - v_{g_{z_1}} \equiv v_{\mathrm{D_{c_I}}} = -\frac{1}{f}\int_{z_1}^{z_2}\left[\frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial z}\frac{\partial p}{\partial x} - \frac{\partial \rho}{\partial x}\frac{\partial p}{\partial z}\right)\right]dz \quad (22\text{-Dc-I-b})$$

where in equation (22-Dc-I-a), $u_{g_{z_2}}$ is eastward component of geostrophic current in lower depth, $u_{g_{z_1}}$ is eastward component of geostrophic current in higher depth, $u_{\mathrm{D_{c_I}}}$is eastward component of first version of dense current, f is Coriolis parameter, z_1 is lower level of the oceanic layer, z_2 is higher level of the oceanic layer, ρ is density of seawater, p is pressure and z is vertical axis of Cartesian coordinates system.
In a similar way, in equation (22-Dc-I-b), $v_{g_{z_2}}$ is northward component of geostrophic current in lower depth, $v_{g_{z_1}}$ is northward component of geostrophic current in higher depth, $v_{\mathrm{D_{c_I}}}$is northward component of first version of dense current, f is Coriolis parameter, z_1 is lower level of the oceanic layer, z_2 is higher level of the oceanic layer, ρ is density of seawater, p is pressure and z is vertical axis of Cartesian coordinates system.
By a view to a careful examination about equation (22-Dc-I-a) and baroclinic vector, i.e., equation (20); we can write:

$$u_{\mathrm{D_{c_I}}} = -\frac{1}{f}\int_{z_1}^{z_2}\mathbb{i}\cdot\left[\frac{1}{\rho^2}(\nabla\rho\times\nabla p)\right]dz \quad (23\text{-Dc-I-a})$$

and by analogy:

$$v_{\mathrm{D_{c_I}}} = -\frac{1}{f}\int_{z_1}^{z_2}\mathbb{j}\cdot\left[\frac{1}{\rho^2}(\nabla\rho\times\nabla p)\right]dz \quad (23\text{-Dc-I-a})$$

Those are other forms of components of first version of dense current. They show the first version of dense current, is fully dependent to baroclinic domain.

Even we can show other particular form of equation (22-Dc-I-a) by Jacobian, same as following:

$$u_{\mathrm{D}_{\mathrm{c}_\mathrm{I}}} = -\frac{1}{f}\int_{z_1}^{z_2}\left(\frac{1}{\rho^2}\, J_u\right) dz \qquad \text{(24-Dc-I-a)}$$

where in equation (24-Dc-I-a), J_u, i.e., Jacobian of u is defined by: [12]

$$J_u \equiv \frac{\partial(\rho.p)}{\partial(y.\ z)} = \begin{vmatrix} \frac{\partial\rho}{\partial y} & \frac{\partial\rho}{\partial z} \\ \frac{\partial p}{\partial y} & \frac{\partial p}{\partial z} \end{vmatrix} = \left(\frac{\partial\rho}{\partial y}\frac{\partial p}{\partial z} - \frac{\partial\rho}{\partial z}\frac{\partial p}{\partial y}\right) \text{(25-Dc-I-a)}$$

where in definition (25-Dc-I-a), J_u is Jacobian for eastward component of first version of dense current.
In similar way, one can show other particular form of equation (22-Dc-I-b) by Jacobian, same as:

$$v_{\mathrm{D}_{\mathrm{c}_\mathrm{I}}} = -\frac{1}{f}\int_{z_1}^{z_2}\left(\frac{1}{\rho^2}\, J_v\right) dz \qquad \text{(24-Dc-I-b)}$$

where in equation (24-Dc-I-b), J_v, i.e., Jacobian of v is defined by:

$$J_v \equiv \frac{\partial(\rho,p)}{\partial(z,x)} = \begin{vmatrix} \frac{\partial\rho}{\partial z} & \frac{\partial\rho}{\partial x} \\ \frac{\partial p}{\partial z} & \frac{\partial p}{\partial x} \end{vmatrix} = \left(\frac{\partial\rho}{\partial z}\frac{\partial p}{\partial x} - \frac{\partial\rho}{\partial x}\frac{\partial p}{\partial z}\right) \text{(25-Dc-I-b)}$$

where in definition (25-Dc-I-b), J_v is Jacobian for northward component of first version of dense current.
The density variation in vertical direction of the oceans is very small, normally only a few parts per thousand. [4] Mostly because, under pycnocline layer, variations of temperature and salinity are less with respect to depth. This fact is depicted in figure 5. [13]

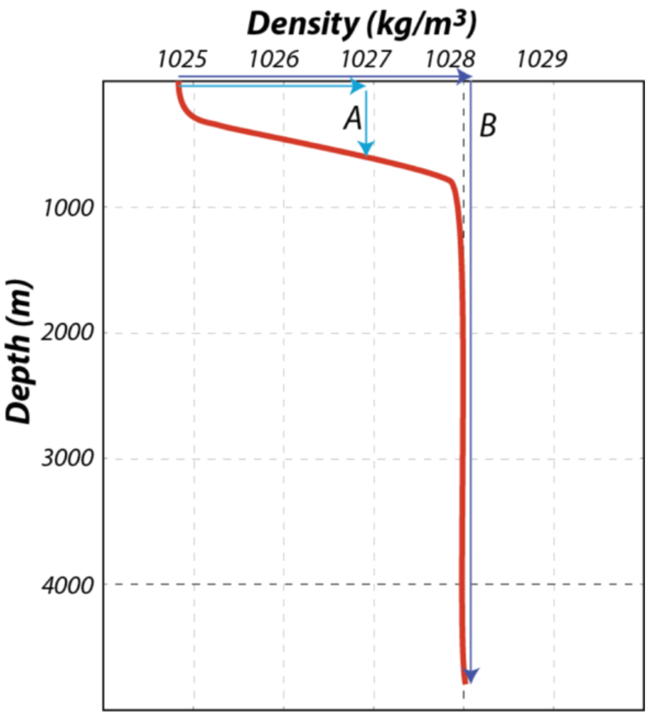

Figure 5. Profile of density with respect to depth. As we see, variation of density with respect to depth can be ignored below pycnocline layer. [13]

In addition, with assumptions, less thickness of the oceanic layer and variations of density are in horizonal directions only; we ignore ${\partial\rho}/{\partial z}$ and remove second terms in right sides of equations (15-a) and (15-b) and now we have:

$$\frac{\partial u_g}{\partial z} = \frac{g}{\rho\, f}\frac{\partial\rho}{\partial y} \qquad \text{(26-a)}$$

and

$$\frac{\partial v_g}{\partial z} = -\frac{g}{\rho\, f}\frac{\partial\rho}{\partial x} \qquad \text{(26-b)}$$

Equation (26-a) shows that variation of eastward component of geostrophic current with respect to z, is proportional to variation of density in northward direction i.e., if we have variation of density in northward direction, we experience variation of eastward component of geostrophic current in vertical direction.
Likewise, Equation (26-b) shows that variation of northward component of geostrophic current with respect to z, is proportional to variation of density in eastward direction i.e., as long as we have variation of density in eastward direction, we experience variation of northward component of geostrophic current in vertical direction.
We can combine equations (26-a) and (26-b) as vector form:

$$\frac{\partial \mathbb{v}_g}{\partial z} = -\frac{g}{\rho\, f}(\mathbb{k} \times \nabla\rho) \qquad \text{(27-Dc-I)}$$

We name equation (27-I-Dc) as *First version of dense current equation.* Where in equation (27-Dc-I), $\mathbb{v}_g$ is geostrophic current, g is acceleration due to gravity, ρ is density of seawater, f is Coriolis parameter and $\mathbb{k}$ stands for vertical unit vector in Cartesian coordinates system.
Equation (27-Dc-I) shows when geostrophic current can change in vertical direction, or when we have dense current that; we have horizontal variation of density. In other word; in the case, ocean medium should be baroclinic.
Furthermore, we can derive *First version of dense current vector* by integration of equation (27-Dc-I) from higher depth z_1 to lower depth z_2 in vertical direction as following: (by pay attention to figure 2)

$$\mathbb{v}_{g(z_2)} - \mathbb{v}_{g(z_1)} \equiv \mathbb{v}_{\mathrm{D}_{\mathrm{c}_\mathrm{I}}} = -\frac{g}{f}\int_{z_1}^{z_2}\frac{1}{\rho}(\mathbb{k} \times \nabla\rho)dz$$
(28-Dc-I)

In equation (28-Dc-I) or equation for first version of dense current vector; $\mathbb{v}_{g(z_2)}$ is geostrophic current in lower depth, $\mathbb{v}_{g(z_1)}$ is geostrophic current in higher depth, $\mathbb{v}_{\mathrm{D}_{\mathrm{c}_\mathrm{I}}}$ is first version of dense current vector, z_1 is lower level of oceanic layer, z_2 is higher level of oceanic layer, ∇ is gradient operator and other parameters defined after equation (27-Dc-I). For equation (28-Dc-I) – that is the equation of first version of dense current vector – we used definition (12).

Eastward and northward components of first version of dense current can be derived by integration of equations (26-a) and (26-b) – similar to deriving equation (28-Dc-I) – directly or one can determine the eastward and northward components of first version of dense current from equation (28-Dc-I) in direct manner:

$$\int_{z_1}^{z_2} \frac{\partial u_g}{\partial z} dz = \int_{z_1}^{z_2} du_g = \int_{z_1}^{z_2} \frac{g}{\rho f} \frac{\partial \rho}{\partial y} dz \qquad \text{(29-Dc-I-a)}$$

By applying integral to (29-Dc-I-a), we get:

$$u_{g_{z_2}} - u_{g_{z_1}} \equiv u_{\mathrm{D_{c_1}}} = \frac{g}{f} \int_{z_1}^{z_2} \frac{1}{\rho} \frac{\partial \rho}{\partial y} dz \qquad \text{(30-Dc-I-a)}$$

and:

$$\int_{z_1}^{z_2} \frac{\partial v_g}{\partial z} dz = \int_{z_1}^{z_2} dv_g = -\int_{z_1}^{z_2} \frac{g}{\rho f} \frac{\partial \rho}{\partial x} dz \qquad \text{(29-Dc-I-b)}$$

and by applying integral to (29-Dc-I-b), we get:

$$v_{g_{z_2}} - v_{g_{z_1}} \equiv v_{\mathrm{D_{c_1}}} = -\frac{g}{f} \int_{z_1}^{z_2} \frac{1}{\rho} \frac{\partial \rho}{\partial x} dz \qquad \text{(30-Dc-I-b)}$$

where equation (30-Dc-I-a) $u_{\mathrm{D_{c_1}}}$ is eastward component of first version of dense current and in equation (30-Dc-I-b) $v_{\mathrm{D_{c_1}}}$ is northward component of first version of dense current. In addition, we used definitions (13-a) and (13-b) for deriving these formulae.

From first version of dense current vector or its components; one can find out that:

A: Whatever we go from pole to equator, dense current becomes stronger[2];

B: If the oceanic layer has more thickness; then dense current becomes more powerful;

C: Whatever oceanic layer is near to oceanic surface, density would be lower and dense current becomes stronger as a result, and

D: If the horizontal gradient of density would be greater, dense current becomes more powerful, because dense current is proportional to the horizontal gradient of density.

In this manner, the first version of dense current vector, i.e., equation (28-Dc-I) shows that: "*First version of dense current flows parallels to Isopycnals, so that, low density is located at the right side of downstream.*" (In the northern hemisphere) This fact is illustrated in figures 6 and 7.

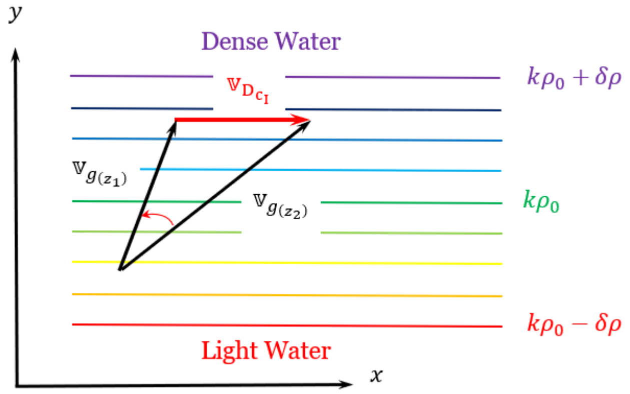

Figure 6. Counterclockwise rotation of geostrophic current with respect to depth (Backing) and light water advection. Coefficient $k = -\frac{g}{f}\langle\frac{1}{\rho}\rangle$

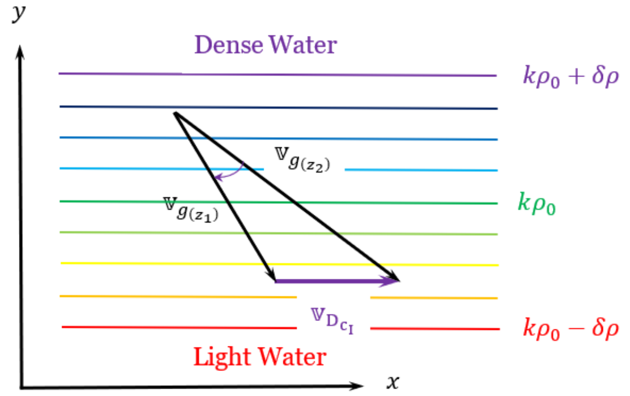

Figure 7. Clockwise turning of geostrophic current with respect to depth; (Veering) and dense water advection. Coefficient $k = -\frac{g}{f}\langle\frac{1}{\rho}\rangle$

In connection with figure 6, counterclockwise turning of geostrophic current with respect to depth (backing) is associated with lower density of seawater advection by geostrophic current in the layer.

In other words, backing of geostrophic current with respect to depth associated with advection of lower density of seawater – that may be from warmer or fresher seawater – in the oceanic layer.

Conversely, as shown in figure 7, clockwise turning of geostrophic current with respect to depth (veering) implies advection of seawater with higher density by geostrophic current in the oceanic layer. In other words, veering of geostrophic current with respect to depth associated with advection of seawater with higher density – that may be from colder or saltier seawater – in the oceanic layer. Also $\langle\frac{1}{\rho}\rangle$ under explanations of figures 6 and 7 is vertical average of $\frac{1}{\rho}$ of oceanic layer[3].

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[2] Use of geostrophic current in tropical regions must be with careful deliberation because geostrophic current in these regions is magnified and especially on equator is meaningless.

[3] There are many procedures about averaging in meteorology and subbranches. Every method for averaging particular parameter or term, is related to position, time and specifications of the domain. All meteorological or oceanographical parameters; decreases logarithmic or semi-logarithmic and increases exponentially or semi-exponentially with respect to height or depth. Therefore, for every case, we need to use special manner for averaging. As

Therefore, it is possible to obtain a reasonable estimate of the horizontal advection of seawater with lower density and its vertical dependence at a given location solely from data on the vertical profile of the current given by current meter. Alternatively, the geostrophic current at any level of the oceanic layer can be estimated from the advection of lower or higher density of seawater field, provided that the geostrophic velocity is known at a single level of the oceanic layer.

Likewise, it is possible to introduce simpler forms of first version of dense current vector and its components. Among of them there are:

$$\mathbb{v}_{\mathrm{D}_{\mathrm{c_I}}} \cong -\frac{g}{f}\langle\frac{1}{\rho}\rangle\int_{z_1}^{z_2}(\mathbb{k}\times\nabla\rho)\,dz \qquad \text{(31-Dc-I)}$$

with eastward component:

$$u_{\mathrm{D}_{\mathrm{c_1}}} \cong \frac{g}{f}\langle\frac{1}{\rho}\rangle\int_{z_1}^{z_2}\frac{\partial\rho}{\partial y}dz \qquad \text{(32-Dc-I-a)}$$

and northward component:

$$v_{\mathrm{D}_{\mathrm{c_1}}} \cong -\frac{g}{f}\langle\frac{1}{\rho}\rangle\int_{z_1}^{z_2}\frac{\partial\rho}{\partial x}dz \qquad \text{(32-Dc-I-b)}$$

Here the angle brackets in equations (31-Dc-I), (32-Dc-I-a) and (32-Dc-I-b) denote vertical average.

Even, one can introduce simpler form of first version of dense current vector and its components, the same as below:

$$\mathbb{v}_{\mathrm{D}_{\mathrm{c_I}}} \cong -\frac{gh}{f}\langle\frac{1}{\rho}\rangle\langle(\mathbb{k}\times\nabla\rho)\rangle \qquad \text{(33-Dc-I)}$$

with eastward component:

$$u_{\mathrm{D}_{\mathrm{c_1}}} \cong \frac{gh}{f}\langle\frac{1}{\rho}\rangle\langle\frac{\partial\rho}{\partial y}\rangle \qquad \text{(34-Dc-I-a)}$$

and northward component:

$$v_{\mathrm{D}_{\mathrm{c_1}}} \cong -\frac{gh}{f}\langle\frac{1}{\rho}\rangle\langle\frac{\partial\rho}{\partial x}\rangle \qquad \text{(34-Dc-I-b)}$$

where in equations (33-Dc-I), (34-Dc-I-a) and (34-Dc-I-b); the thickness of the layer is:

$$h = z_2 - z_1 \qquad (35)$$

4.4. Relationship between dense current and geopotential thickness

We can examine the relationship between dense current and geopotential thickness. Consider equation (26-a):

$$\frac{\partial u_g}{\partial z} = \frac{g}{\rho\, f}\frac{\partial\rho}{\partial y} \qquad \text{(26-a)}$$

and remember definition of geopotential in Cartesian coordinates system, that is [1], [5]:

$$\Phi = gz \qquad (36)$$

where Φ is geopotential. Geopotential is multiplication of height and acceleration due to gravity in Cartesian coordinates system. One can rewrite equation (26-a) same as:

$$\frac{\partial u_g}{\partial z}dz = \frac{g}{\rho\, f}\frac{\partial\rho}{\partial y}dz \qquad (37)$$

or whit due to the fact that we take into account the acceleration due to gravity as constant parameter for various meteorological subject, we get:

$$du_g = \frac{1}{\rho\, f}\frac{\partial\rho}{\partial y}g dz \qquad (38)$$

or by applying definition of geopotential via equation (36) and by definition of geopotential differential that is: [5]

$$d\Phi = g dz \qquad (39)$$

and replacing in equation (38), one can get:

$$du_g = \frac{1}{\rho\, f}\frac{\partial\rho}{\partial y}d\Phi \qquad \text{(40-a)}$$

Analogically; by same mathematical manipulation on equation (26-b), we get:

$$dv_g = -\frac{1}{\rho\, f}\frac{\partial\rho}{\partial x}d\Phi \qquad \text{(40-b)}$$

By combining equations (40-a) and (40-b) as vector form, following result cat be got:

$$d\mathbb{v}_g = -\frac{1}{\rho\, f}(\mathbb{k}\times\nabla\rho)\,d\Phi \qquad (41)$$

where in equation (41), $\mathbb{v}_g$ is geostrophic current, ρ is seawater density, f is Coriolis parameter, $\mathbb{k}$ is vertical unit vector in Cartesian coordinates system, ∇ is gradient operator and Φ stands for geopotential.

By pay attention to figure 2; if we integrate equation (41) in vertical direction from lower level z_1 with geopotential $\Phi_1$to higher level z_2 with geopotential Φ_2 same following:

$$\int_{z_1}^{z_2} d\mathbb{v}_g = -\frac{1}{f}\int_{z_1}^{z_2}\frac{1}{\rho}(\mathbb{k}\times\nabla\rho)\,d\Phi \qquad \text{(42-Dc-II)}$$

yields, <u>*Second version of dense current vector*</u>, that is:

$$\mathbb{v}_{g_{z_2}} - \mathbb{v}_{g_{z_1}} \equiv \mathbb{v}_{\mathrm{D}_{\mathrm{c_{II}}}} = -\frac{1}{f}\int_{z_1}^{z_2}\frac{1}{\rho}(\mathbb{k}\times\nabla\rho)\,d\Phi \qquad \text{(28-Dc-II)}$$

In equation (28-Dc-II), $\mathbb{v}_{g_{z_2}}$ is geostrophic current in lower depth, $\mathbb{v}_{g_{z_1}}$ is geostrophic current in higher

a simple example, if we consider a layer of seawater between 200- and 300-meters depth; the easiest way is using linear averaging, i.e., adding the value of parameters or terms of above and below levels of the layer and dividing the result by 2.

depth, $\mathbb{v}_{\mathrm{D_{c_{II}}}}$ is the second version of dense current vector, f is Coriolis parameter, z_1 is depth of lower level of oceanic layer, z_2 is depth of higher level of oceanic layer, ρ is density of seawater, $\mathbb{k}$ is vertical unit vector in Cartesian coordinates system, ∇ is gradient operator and Φ is geopotential. Equation (28-Dc-II) is the second version of equation of dense current vector and we have used definition (12) for extracting it.

Eastward and northward components of second version of dense current can be derived by integration of equations (40-a) and (40-b) – similar to deriving equation (28-Dc-II) – directly, or one can determine the eastward and northward components of second version of dense current from equation (28-Dc-II) in direct manner:

$$\int_{z_1}^{z_2} du_g = \frac{1}{f}\int_{z_1}^{z_2} \frac{1}{\rho}\frac{\partial \rho}{\partial y} d\Phi \qquad \text{(43-Dc-II-a)}$$

and:

$$\int_{z_1}^{z_2} dv_g = -\frac{1}{f}\int_{z_1}^{z_2} \frac{1}{\rho}\frac{\partial \rho}{\partial x} d\Phi \qquad \text{(43-Dc-II-b)}$$

Calculation integrals of equations (43-Dc-II-a) and (43-Dc-II-b) yields:

$$u_{g_{z_2}} - u_{g_{z_1}} \equiv u_{\mathrm{D_{c_{II}}}} = \frac{1}{f}\int_{z_1}^{z_2} \frac{1}{\rho}\frac{\partial \rho}{\partial y} d\Phi \qquad \text{(30-Dc-II-a)}$$

and:

$$v_{g_{z_2}} - v_{g_{z_1}} \equiv v_{\mathrm{D_{c_{II}}}} = -\frac{1}{f}\int_{z_1}^{z_2} \frac{1}{\rho}\frac{\partial \rho}{\partial x} d\Phi \qquad \text{(30-Dc-II-b)}$$

where in equation (30-Dc-II-a), $u_{g_{z_2}}$is eastward component of geostrophic current on higher level of oceanic layer, $u_{g_{z_1}}$is eastward component of geostrophic current on lower level of oceanic layer, $u_{\mathrm{D_{c_{II}}}}$ is eastward component of second version of dense current and we have used definition (13-a) for deriving it. And in equation (30-Dc-II-b), $v_{g_{z_2}}$is northward component of geostrophic current on higher level of oceanic layer, $v_{g_{z_1}}$is northward component of geostrophic current on lower level of oceanic layer, $v_{\mathrm{D_{c_{II}}}}$ is northward component of second version of dense current and we have used definition (13-b) for extracting this equation.

All specifications, properties and figures of the second version of dense current are the same as the first version of dense current approximately.

Also, it is possible to introduce simple forms of the second version of dense current vector and its components. Among of them there are:

$$\mathbb{v}_{\mathrm{D_{c_{II}}}} \cong -\frac{1}{f}\left\langle\frac{1}{\rho}\right\rangle \int_{z_1}^{z_2} (\mathbb{k} \times \nabla\rho)\, d\Phi \qquad \text{(31-Dc-II)}$$

with eastward component:

$$u_{\mathrm{D_{c_{II}}}} \cong \frac{1}{f}\left\langle\frac{1}{\rho}\right\rangle \int_{z_1}^{z_2} \frac{\partial \rho}{\partial y} d\Phi \qquad \text{(32-Dc-II-a)}$$

and northward component:

$$v_{\mathrm{D_{c_{II}}}} \cong -\frac{1}{f}\left\langle\frac{1}{\rho}\right\rangle \int_{z_1}^{z_2} \frac{\partial \rho}{\partial x} d\Phi \qquad \text{(32-Dc-II-b)}$$

Here the angle brackets in equations (31-Dc-II), (32-Dc-II-a) and (32-Dc-II-b) denote a vertical average.

Even, one can introduces simpler form of second version of dense current vector and its components the same as below:

$$\mathbb{v}_{\mathrm{D_{c_{II}}}} \cong -\frac{1}{f}\left\langle\frac{1}{\rho}\right\rangle \langle(\mathbb{k} \times \nabla\rho)\rangle(\delta\Phi) \qquad \text{(33-Dc-II)}$$

with eastward component:

$$u_{\mathrm{D_{c_{II}}}} \cong \frac{1}{f}\left\langle\frac{1}{\rho}\right\rangle \left\langle\frac{\partial \rho}{\partial y}\right\rangle (\delta\Phi) \qquad \text{(34-Dc-II-a)}$$

and northward component:

$$v_{\mathrm{D_{c_{II}}}} \cong -\frac{1}{f}\left\langle\frac{1}{\rho}\right\rangle \left\langle\frac{\partial \rho}{\partial x}\right\rangle (\delta\Phi) \qquad \text{(34-Dc-II-b)}$$

where in equations (33-Dc-II), (34-Dc-II-a) and (34-Dc-II-b); the geopotential thickness of the layer is:

$$\delta\Phi = \Phi_2 - \Phi_1 \qquad (44)$$

4.5. Other view to the relationship between dense current and geopotential thickness

Geostrophic approximation in pressure coordinates system[4], is introduced in many papers or textbooks in the following manner: [5]

$$f\mathbb{v}_g = \mathbb{k}_p \times \nabla_p \Phi \qquad (45)$$

where in equation (45), f is Coriolis parameter, $\mathbb{v}_g$ is geostrophic velocity, $\mathbb{k}_p$ is unit vector in pressure coordinates system, ∇_p is gradient operator in pressure coordinates system and Φ is geopotential in this coordinates system.

Writing components of equation (45) yields:

$$fu_g = -\frac{\partial \Phi}{\partial y} \qquad \text{(46-a)}$$

and:

$$fv_g = \frac{\partial \Phi}{\partial x} \qquad \text{(46-b)}$$

[4] In this work; whenever we refer to *"pressure coordinates system"* our purpose is "*Cartesian coordinates system with pressure as vertical coordinate*". Note this coordinates system is "*Left-handed system*" and some textbooks referred to it as "*Isobaric coordinates system*"

Calculation of velocity components has following results:

$$u_g = -\frac{1}{f}\frac{\partial \Phi}{\partial y} \quad (47\text{-a})$$

and:

$$v_g = \frac{1}{f}\frac{\partial \Phi}{\partial x} \quad (47\text{-b})$$

where in equations (46-a) and (47-a), u_g is eastward component of geostrophic current and in equations (46-b) and (47-b), v_g is northward component of geostrophic current. Likewise, geostrophic current is:

$$\mathbb{v}_g = \frac{1}{f}\mathbb{k}_p \times \nabla_p \Phi \quad (48)$$

where in equation (48), $\mathbb{v}_g$ is geostrophic current, f is Coriolis parameter, $\mathbb{k}_p$ is unit vector in pressure coordinates system, ∇_p is gradient operator in pressure coordinates system and Φ is geopotential in this coordinates system.
Equation (48) shows that magnitude of geostrophic current is proportional to gradient of geopotential and is parallel to isopleths of geopotential on isobaric surface. Figure 8 shows geostrophic current in pressure coordinates system.

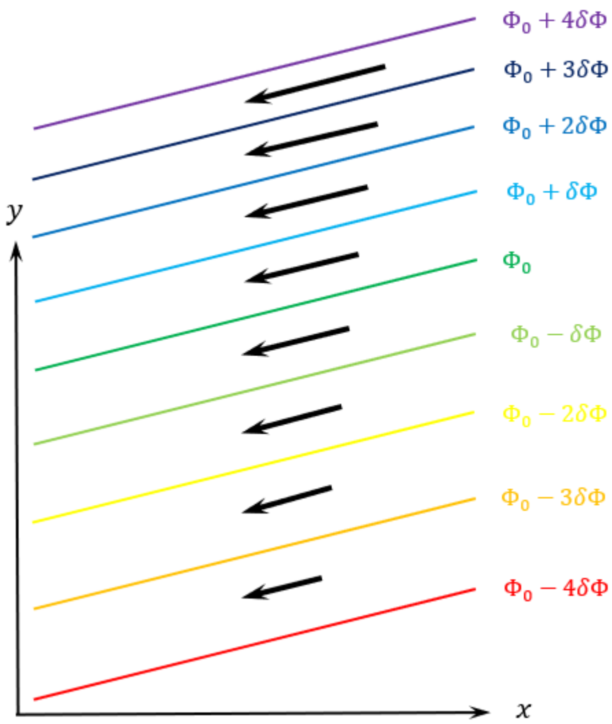

Figure 8. Geostrophic current in pressure coordinates system; parallel to geopotentials.

If we select equation (7):

$$dp = -\rho g dz \quad (7)$$

and assume oceanic layer that lower level has zero height and pressure p and higher level of it has z height and zero pressure, then integrate equation (7) from lower level of this layer to higher level of it, i.e.:

$$\int_p^0 dp = -\int_0^z \rho g dz \quad (49)$$

Calculation of above integrals yields:

$$p = \rho_0 g z \quad (50)$$

where in equation (50) ρ_0 is average density of the layer. By substituting of gdz from equation (36), i.e., Φ, in equation (50) we get:

$$p = \rho_0 \Phi \quad (51)$$

or:

$$\frac{p}{\rho_0} = \Phi \quad (52)$$

Equation (52) shows, if pressure would be constant – same as horizontal moving in pressure coordinates system – geopotential will be a function of density. i.e.,

$$\Phi = \Phi(\rho) \quad (53)$$

and according to the equation of state, i.e., equation (11)

$$\rho = \rho(s.T.p) \quad (11)$$

therefore:

$$[\Phi = \Phi(\rho) \ \wedge \ \rho = \rho(s.T.p)] \Longrightarrow \Phi = \Phi(s.T.p) \quad (54)$$

So, if horizontal temperature gradient or horizontal salinity gradient varies on pressure surface in oceanic environment; then geopotential gradient varies and geostrophic current varies with respect to depth.
Differentiating from equations (47-a) and (47-b) with respect to p yields:

$$\frac{\partial u_g}{\partial p} = -\frac{1}{f}\frac{\partial}{\partial p}\frac{\partial \Phi}{\partial y} = -\frac{1}{f}\frac{\partial}{\partial y}\frac{\partial \Phi}{\partial p} \quad (55\text{-a})$$

and:

$$\frac{\partial v_g}{\partial p} = \frac{1}{f}\frac{\partial}{\partial p}\frac{\partial \Phi}{\partial x} = \frac{1}{f}\frac{\partial}{\partial x}\frac{\partial \Phi}{\partial p} \quad (55\text{-b})$$

those can be written as:

$$\frac{\partial u_g}{\partial p} dp = -\frac{1}{f}\frac{\partial}{\partial y}\frac{\partial \Phi}{\partial p} dp \quad (56\text{-a})$$

and:

$$\frac{\partial v_g}{\partial p} dp = \frac{1}{f}\frac{\partial}{\partial x}\frac{\partial \Phi}{\partial p} dp \quad (56\text{-b})$$

Also, equations (56-a) and (56-b) can be written as following forms:

$$du_g = -\frac{1}{f}\frac{\partial}{\partial y} d\Phi \quad (57\text{-a})$$

and:

$$dv_g = \frac{1}{f}\frac{\partial}{\partial x}d\Phi \tag{57-b}$$

Now, considering figure 2, and integrating equation (57-a) from lower level of the oceanic layer with pressure p_1 and geopotential Φ_1, to higher level of the oceanic layer with pressure p_2 and geopotential Φ_2 yields:

$$\int_{p_1}^{p_2} du_g = -\frac{1}{f}\int_{\Phi_1}^{\Phi_2}\frac{\partial}{\partial y}d\Phi \tag{58-a}$$

After applying integration, we get:

$$u_{g_{p_2}} - u_{g_{p_1}} \equiv u_{\mathrm{D_{c_{III}}}} = -\frac{1}{f}\frac{\partial}{\partial y}(\Phi_2 - \Phi_1) \tag{30-Dc-III-a}$$

In equation (30-Dc-III-a), $u_{\mathrm{D_{c_{III}}}}$ is eastward component of third version of dense current and we used definition (13-a) for deriving this equation.
Equation (30-Dc-III-a) shows that, if northward going causes to reduction of oceanic thickness; then eastward component of third version of dense current is positive.
Likewise; by fixing the mind on figure 2, and integrating equation (57-b) from lower level of the oceanic layer with pressure p_1 and geopotential Φ_1, to higher level of the oceanic layer with pressure p_2 and geopotential Φ_2 yields:

$$\int_{p_1}^{p_2} dv_g = \frac{1}{f}\int_{\Phi_1}^{\Phi_2}\frac{\partial}{\partial x}d\Phi \tag{58-b}$$

After applying integration, we get:

$$v_{g_{p_2}} - v_{g_{p_1}} \equiv v_{\mathrm{D_{c_{III}}}} = \frac{1}{f}\frac{\partial}{\partial x}(\Phi_2 - \Phi_1) \tag{30-Dc-III-b}$$

In equation (30-Dc-III-b), $v_{\mathrm{D_{c_{III}}}}$ is northward component of third version of dense current and we used definition (13-b) for deriving this equation.
Equation (30-Dc-III-b) shows that, if eastward going causes to increasing of oceanic thickness; then northward component of third version of dense current is positive.
And, if we combine equations (30-Dc-III-a) and (30-Dc-III-b) as vector form, we get:

$$\mathbb{v}_{\mathrm{D_{c_{III}}}} = f^{-1}\mathbb{k} \times \nabla_p(\Phi_2 - \Phi_1) \tag{28-Dc-III}$$

where, in equation (28-Dc-III), $\mathbb{v}_{\mathrm{D_{c_{III}}}}$ is: _Third version of dense current vector_, f is Coriolis parameter, $\mathbb{k}$ is vertical unit vector in pressure coordinates system, ∇_p is gradient operator in pressure coordinates system, Φ_2 is geopotential of higher level of the oceanic layer, Φ_1 is geopotential of lower level of the oceanic layer and $(\Phi_2 - \Phi_1)$ is geopotential thickness of the oceanic layer. The general type of third version of dense current is depicted in figure 9.

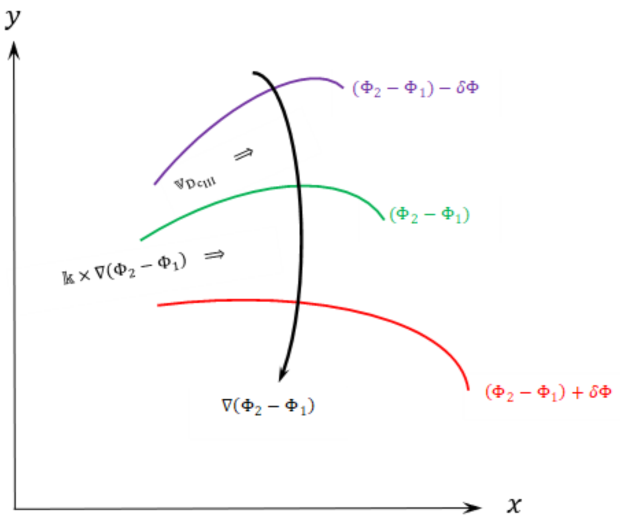

Figure 9. Typical form of third version of dense current.

From third version of dense current vector or its components; one can find out that:
A: Whatever we go from pole to equator, third version of dense current becomes stronger (with pay attention to footnote No. 2) and
B: If the horizontal gradient of geopotential thickness of the oceanic layer would be greater; then third version of the dense current becomes more powerful, because this version of dense current is proportional to the horizontal gradient of geopotential thickness.
In this manner, the third version of dense current vector, i.e., equation (28-Dc-III) shows that: "*Third version of dense current flows parallels to isopleths of geopotential thickness of the oceanic layer so that, more thickness of the layer is located at the right side of downstream.*" (In the northern hemisphere) This is illustrated in Figures 10 and 11.

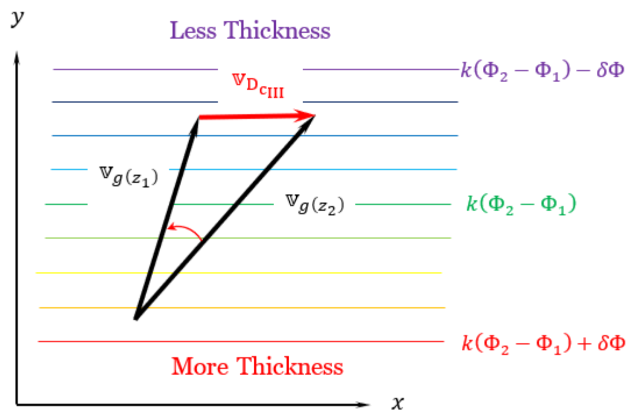

Figure 10. Counterclockwise rotation of geostrophic current with respect to depth (Backing) and more thickness of oceanic layer advection. And $k = \frac{1}{f}$

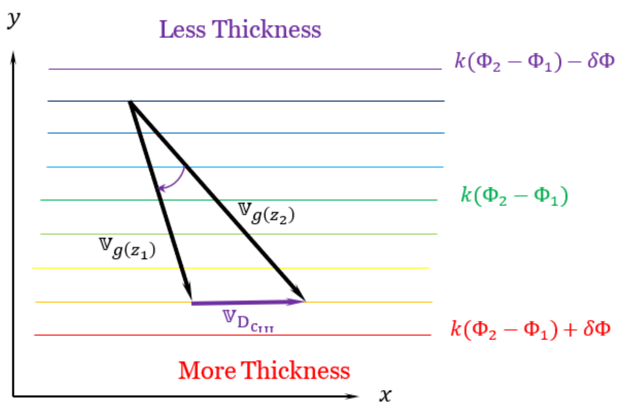

Figure 11. Clockwise turning of geostrophic current with respect to depth (Veering) and less thickness of oceanic layer advection. And $k = \frac{1}{f}$

In connection with figure 10; counterclockwise turning of geostrophic current with respect to depth (backing) is associated with more thickness of oceanic layer advection by geostrophic current in the layer.
Conversely, as shown in figure 11, clockwise rotation of geostrophic current with respect to depth (veering) implies advection of less thickness of oceanic layer.
Therefore, it is possible to obtain a reasonable estimate of the horizontal advection of oceanic layer thickness and its vertical dependence at a given location solely from data on the vertical profile of the current given by current meter. Alternatively, the geostrophic current at any level of the oceanic layer can be estimated from the advection of less or more thickness of oceanic layer, provided that the geostrophic velocity is known at other level of that oceanic layer.

5. Results and Discussion

All versions of the dense current equations are extremely useful diagnostic tools, which is often used to check analyses of the observed current field for consistency.
It can also be used to estimate the mean horizontal oceanic seawater density, and thickness advections in an oceanic layer as shown in Figures 6, 7, 10 and 11 respectively.
Dense current is the struggle of the oceanic medium to return thermodynamic equilibrium and complete the dynamic cycle of ocean. This movement begins from the fact that solar radiation disturbs the thermodynamic equilibrium of oceanic ambience, resulting production of horizontal gradient of density. Horizontal gradient of density produces horizontal gradient of potential energy and in turn, this condition forces ocean to generate horizontal gradient of pressure and finally, it causes to start current for returning thermodynamic equilibrium of the ocean.
Concerning various insolation and non-uniform transfer of diffusion of heat and salinity in the different layers of the ocean; horizontal gradients of density are not same in the oceanic layers and current velocities cannot be the same at oceanic layers. Therefore, this phenomenon produces dense current that is effort of ocean to reduce horizontal gradient of density and in turn, reducing dense current speed. By continuous reduction of dense current speed, thermodynamic disequilibrium of ocean weakens and weakens, until returning the thermodynamic equilibrium of the ocean. If we assume there will be no more solar radiation, finally the ocean current will be disappeared in the presence of friction.
So, dense current that produced by baroclinic domain, is main response of the oceanic environment for returning thermodynamic equilibrium again so that, oceanic currents including geostrophic currents transfer high density seawater to location of low-density seawater and analogically; transfer high density seawater to location of low-density seawater; as well as advects more thickness oceanic layer toward less thickness oceanic layer and vice versa, in the condition. In addition, dense currents have important role for diffusing salt in oceans for the purpose of balancing salinity in oceans.
Also, dense current vector equations, equations of eastward component of dense current and equations of northward component of dense current show:
5 – 1 – At any time that density varies in horizontal direction i.e.; if there is baroclinic medium; there is dense current too;
5 – 2 – Whatever we approach to tropical latitudes, dense current will be powered (with pay attention to footnote No. 2);
5 – 3 – Whatever we close to ocean surface – for reason of light density - dense current has more force;
5 – 4 – If there is high horizontal gradient of density; dense current has more strength and
5 – 5 – If there is more horizontal gradient of thickness of the oceanic layer; then dense current has more power.
Likewise, the proposition of dense current, that is propounded with these kind configurations; can help to better understanding of ocean dynamics. Furthermore, knowing the advection of the heavy seawater density, light seawater density or advection of the thickness of the oceanic layer with exclusive specifications; can help to improve oceanic forecasting. Although, we referred to the variation of the geostrophic current with respect to depth here; but this proposition is valid for the variation of real oceanic current in vertical direction with some modifications that needs separate discussion.
And study of dense current can enlighten deep sea dynamics and helps to better understanding climate of deep oceans.

6. Conclusions

Many phenomena and mechanisms have essential roles for transfer and diffusion of arriving solar radiation from tropical regions to high latitudes in northern hemisphere and low latitudes in southern hemisphere.

The most important of these mechanisms are: winds, ocean currents, sensible heat, latent heat, monsoon phenomenon (as other type of latent heat transfer), tropical cyclones, meridional overturning circulation, Rossby waves and Antarctic circumpolar current.

Atmospheric movements (winds) and oceanic movements (currents) are some parts of these mechanisms. Even different shear of them (movements) including their vertical shear; have basic roles in the subject.

Our goals in this article were familiarizing with oceanic efforts for transfer of sensible heat via dense current that is vertical shear of geostrophic current in baroclinic ocean.

It is necessary to note the basic point. The atmosphere and oceanic waters are baroclinic. And the theory of barotropic ambience – same as geostrophic balance – is acceptable for simplification of meteorological and oceanic analyses.

However, baroclinic environment, geostrophic current and variation of it with respect to depth, definition of dense current, deriving of dense current vector in Cartesian coordinates system, the other view to derive equation of dense current, relation between dense current and geopotential thickness, other view of relation between dense current and geopotential thickness, have discussed expanded upon the article.

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