INTERNATIONAL JOURNAL OF
COASTAL, OFFSHORE & ENVIRONMENTAL ENGINEERING IJCOE Vol.8/No. 2/Spring 2023 (21-39)

Available online at: www. Ijcoe.org

A New Look at the Vertical Shear of the Geostrophic Wind Part II: Thermal Wind and Moist Wind

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ARTICLE INFO

Article History:
Received: 23 Oct. 2022
Accepted: 21 May. 2023

Keywords:
Baroclinic medium
Geostrophic wind
Dense wind
Thermal wind
Moist wind

ABSTRACT

We can divide atmosphere into two mediums, barotropic and baroclinic. Due to horizontal gradient of density, baroclinic medium causes to produce various horizontal gradient of pressure with respect to height and implies various horizontal velocities at different layers of the atmosphere. Therefore; geostrophic wind varies with respect to height in this medium.

The horizontal gradient of density not only would produce by horizontal gradient of temperature, but also by horizontal gradient of humidity or combination of both.

If horizontal gradient of density would be by both horizontal gradient of temperature and horizontal gradient of humidity – as they are existing in natural air – in the case; vectorial difference of geostrophic wind with respect to height is; *dense wind*.

If horizontal gradient of density is related to gradient of temperature solely; vectorial difference between geostrophic wind from top level and bottom level of the layer is; *thermal wind*.

And if horizontal gradient of density is solely related to gradient of specific humidity; vectorial difference between geostrophic wind from top level and bottom level of the layer is; *moist wind*.

The purpose of this paper is confirmation of three versions of dense wind, introduction five particular types of thermal wind and present two prominent types of moist wind in natural medium of air. Formulae related to each type are derived and every one of them, represents effects of one type of variation of geostrophic wind with respect to height.

1. Introduction

In part I of "*A New Look at the Vertical Shear of the Geostrophic Wind: Dense Wind*" we referred to some historical experiments about vertical shear of the geostrophic wind and number of basic ideas in connection with definition and deriving formulae to describe "*Dense Wind*". [1]

We pointed out that various heat flux or humidity flux can lead to produce baroclinic atmosphere and in turn; variation of geostrophic wind in vertical direction. Furthermore, there is direct interaction between heat or humidity flux with wind shear.

In connection with the subject and as one feature of the role of heat flux to wind shear; Kim et. al. in their study through large-eddy simulation found out, that constant kinematic heat flux of $0.05\ \mathrm{Jm^{-2}s^{-1}}$causes for varying geostrophic wind speeds from 5 to $15\mathrm{ms^{-1}}$. Heat flux profiles show that the maximum entrainment heat flux as a fraction of the surface heat flux, increases from 0.13 to 0.30 in magnitude with increasing wind shear. The thickness of the entrainment layer, relative to the depth of the well-mixed layer, increases substantially from 0.36 to 0.73 with increasing wind shear. [2]

Now, some important points of part I are as follows:

1 – 1 – Vertical shear of the wind was research work of meteorologists in previous century. Among of them, we referred to works of Charnock et al. [3], Carlstead [4], Estoque [5] and Foster and Levy [6].

1 – 2 – Also we mentioned that all researchers those have been worked on the variation of geostrophic wind with respect to height, had two common ideas. They have been called difference between two geostrophic wind vectors at two pressure levels; the *thermal wind* as the first idea. In addition, they have been assumed the atmosphere is dry air, as a second idea.

Especially, this subject with same hypotheses, has been used in dynamic meteorology's text books, that is to say, in introducing of thermal wind, they assumed atmosphere is dry and derived formulae related to the subject in this case; although this assumption used for simplicity of the work. For instance, the subject is written in Hess's text book [7], Gill's text book [8], Dutton's text book [9], Holton and Hakim's text book [10] and in other dynamic meteorology text books. In addition; thermal wind has an entrée in *Glossary of Meteorology* [11] with the same descriptions. [1]

1 – 3 – Moreover, after some description of geostrophic wind in pressure coordinates system [1]; reason of variation of geostrophic wind with respect to height showed by logical argument and figure 1.

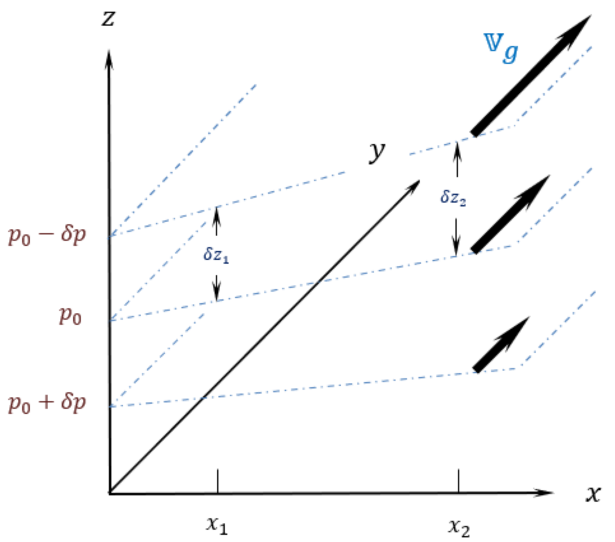

Figure 1. Relationship between vertical shear of the geostrophic wind and horizontal height gradients. (Note that $0 < \delta p$) [10]

Afterward; we defined "*Dense Wind*" as "*<u>vectorial difference of geostrophic wind vector at upper level and geostrophic wind vector at lower level</u>*" (of the atmospheric layer), that is:

$$\mathbb{v}_{\mathrm{D}} \equiv \mathbb{v}_{g(p_2)} - \mathbb{v}_{g(p_1)} \tag{1-D}$$

where in Equation (1) $\mathbb{v}_{\mathrm{D}}$ stands for dense wind vector, $\mathbb{v}_g$ is geostrophic wind, and subscripts p_1 and p_2 refer to pressure levels in the manner that level p_2 has more height than level p_1, i.e., $p_2 < p_1$.

According to definition (1); eastward and northward components of dense wind can be shown as following:

$$u_{\mathrm{D}} = u_{g(p_2)} - u_{g(p_1)} \tag{2-D-a}$$

and

$$v_{\mathrm{D}} = v_{g(p_2)} - v_{g(p_1)} \tag{2-D-b}$$

where u_{D} is eastward component of dense wind, u_g is eastward component of geostrophic wind, v_{D} is northward component of dense wind, v_g is northward component of geostrophic wind and subscripts p_1 and p_2 refer to pressure levels in the manner that level p_2 has more height than level p_1, i.e., $p_2 < p_1$.

Typical layer of atmospheric system is shown in figure 2.

Despite its name, dense wind, while a vector, is not a true wind. Instead, it is a geostrophic wind shear, representing the change of wind with respect to height, causing some advections. [1]

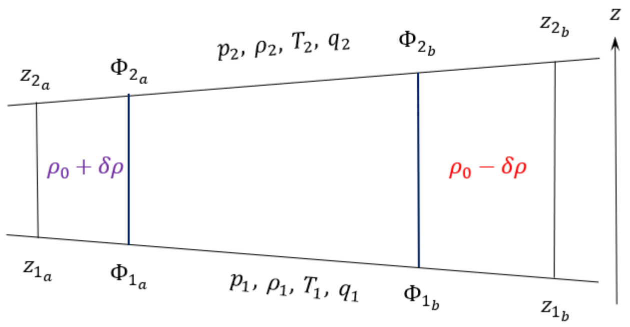

Figure 2. Typical layer of atmospheric system.

1 – 4 – Furthermore, with using a number of formulae from a few text books same as Curry and Webster [12], Iribarne and Godson [13] and Haltiner and Williams [14]; we derived three versions of dense wind's equations, vectors and components. [1]

1 – 5 – For the first version of dense wind; dense wind equation is:

$$\frac{\partial \mathbb{v}_g}{\partial p} = \frac{1}{f(\rho_{\mathrm{M}})^2}\left(\mathbb{k}_p \times \nabla_p \rho_{\mathrm{M}}\right) \tag{3-D-I}$$

and first version of dense wind vector is:

$$\mathbb{v}_{\mathrm{D_I}} = \frac{1}{f}\int_{p_1}^{p_2}\left[\frac{1}{(\rho_{\mathrm{M}})^2}\left(\mathbb{k}_p \times \nabla_p \rho_{\mathrm{M}}\right)\right] dp \tag{4-D-I}$$

where in equations (3-D-I) and (4-D-I), $\mathbb{v}_g$ is geostrophic wind, p is pressure, f is Coriolis parameter, ρ_{M} stands for density of humid air, $\mathbb{k}_p$ is vertical unit vector in pressure coordinates system, ∇_p is gradient operator in pressure coordinates system, $\mathbb{v}_{\mathrm{D_I}}$ stands for first version of dense wind vector, p_1 is atmospheric pressure at lower level of the atmospheric layer and p_2 is atmospheric pressure at upper level of the atmospheric layer.

Eastward and northward components of first version of dense wind can be derived from equation (4-D-I) directly:

$$u_{\mathrm{D_I}} = -\frac{1}{f}\int_{p_1}^{p_2}\left(\frac{1}{(\rho_{\mathrm{M}})^2}\frac{\partial \rho_{\mathrm{M}}}{\partial y}\right) dp \tag{5-D-I-a}$$

[1] In the part I and in this paper (part II); whenever we refer to "*pressure coordinates system*" our purpose is "*Cartesian coordinates system with pressure as vertical coordinate*". Note this coordinates system is "*Left-handed system*".

and

$$v_{\mathrm{D_I}} = \frac{1}{f}\int_{p_1}^{p_2}\left(\frac{1}{(\rho_{\mathrm{M}})^2}\frac{\partial \rho_{\mathrm{M}}}{\partial x}\right)dp \qquad \text{(5-D-I-b)}$$

where in equation (5-D-I-a) $u_{\mathrm{D_I}}$ is eastward component of first version of dense wind and y is northward axis of pressure coordinates system. In addition; in equation (5-D-I-b) $v_{\mathrm{D_I}}$ stands for northward component of first version of dense wind and x is eastward axis of pressure coordinates system.

In this case; clockwise rotation of geostrophic wind with respect to height, associated with light air advection and counterclockwise turning of geostrophic wind with respect to height connected with dense air advection. [1]

1 – 6 – For the second version of dense wind; dense wind equation is:

$$\frac{\partial \mathbb{v}_g}{\partial \ln p} = -\frac{R_d}{f}\left(\mathbb{k}_p \times \nabla_p T_v\right) \qquad \text{(3-D-II)}$$

and second version of dense wind vector is:

$$\mathbb{v}_{\mathrm{D_{II}}} = -\frac{R_d}{f}\int_{p_1}^{p_2}\left(\mathbb{k}_p \times \nabla_p T_v\right)d\ln p \qquad \text{(4-D-II)}$$

where in equations (3-D-II) and (4-D-II); R_d is gas constant for dry air, T_v stand for virtual temperature, $\mathbb{v}_{\mathrm{D_{II}}}$ is second version of dense wind vector and other symbols are defined under 1 – 5.

Eastward and northward components of second version of dense wind can be derived from equation (4-D-II) directly as follows:

$$u_{\mathrm{D_{II}}} = \frac{R_d}{f}\int_{p_1}^{p_2}\frac{\partial T_{\mathrm{v}}}{\partial y}d\ln p \qquad \text{(5-D-II-a)}$$

and

$$v_{\mathrm{D_{II}}} = -\frac{R_d}{f}\int_{p_1}^{p_2}\frac{\partial T_{\mathrm{v}}}{\partial x}d\ln p \qquad \text{(5-D-II-b)}$$

where in equation (5-D-II-a), $u_{\mathrm{D_{II}}}$ is eastward component of second version of dense wind, y is northward axis in pressure coordinates system and in equation (5-D-II-b), $v_{\mathrm{D_{II}}}$ is northward component of second version of dense wind and x is eastward axis of pressure coordinates system. In the case of second version of dense wind; clockwise rotation of geostrophic wind with respect to height, associated with warmer or more humid air advection and counterclockwise turning of geostrophic wind with respect to height connected with colder or less humid air advection. [1]

1 – 7 – For the third version of dense wind; dense wind equation is:

$$\frac{\partial \mathbb{v}_g}{\partial p} = \frac{1}{f}\mathbb{k}_p \times \nabla_p\left(\frac{\partial \Phi}{\partial p}\right) \qquad \text{(3-D-III)}$$

and third version of dense wind vector is:

$$\mathbb{v}_{\mathrm{D_{III}}} = \frac{1}{f}\mathbb{k}_p \times \nabla_p(\Phi_2 - \Phi_1) \qquad \text{(4-D-III)}$$

where in equation (3-D-III) Φ is geopotential and in equation (4-D-III) $\mathbb{v}_{\mathrm{D_{III}}}$ stands for third version of dense wind vector, Φ_2 refers to geopotential of upper level and Φ_1 points to geopotential of lower level of the atmospheric layer and other symbols are defined under 1 – 5.

Eastward and northward components of third version of dense wind can be derived from equation (4-D-III) directly as follows:

$$u_{\mathrm{D_{III}}} = -\frac{1}{f}\frac{\partial}{\partial y}(\Phi_2 - \Phi_1) \qquad \text{(5-D-III-a)}$$

and

$$v_{\mathrm{D_{III}}} = \frac{1}{f}\frac{\partial}{\partial x}(\Phi_2 - \Phi_1) \qquad \text{(5-D-III-a)}$$

where in equation (5-D-III-a), $u_{\mathrm{D_{III}}}$ is eastward component of third version of dense wind, y is northward axis in pressure coordinates system and in equation (5-D-III-b), $v_{\mathrm{D_{III}}}$ is northward component of third version of dense wind and x is eastward axis of pressure coordinates system. Finally, in the case of third version of dense wind; clockwise rotation of geostrophic wind with respect to height, associated with advection of atmospheric thicker layer and counterclockwise turning of geostrophic wind with respect to height connected with advection of less thickness layer of atmosphere. [1]

Following part I; in this part; we focus on special cases of dense wind.

2. Special cases of Dense Wind

In part I of "*A New Look at the Vertical Shear of Geostrophic Wind: Dense Wind*" we referred to virtual temperature as follows:

$$T_v = (1 + 0.608q)T \qquad (6)$$

where in equation (6) q is specific humidity and T stands for temperature. [12]

Choosing equivalent of T_v from equation (6) and substituting into equation (4-D-II) yields:

$$\mathbb{v}_{\mathrm{D_{II}}} = -\frac{R_d}{f}\int_{p_1}^{p_2}\left[\mathbb{k}_p \times \nabla_p(1 + 0.608q)T\right]d\ln p \qquad \text{(7-D-II)}$$

Equation (7-D-II) is dense wind vector in terms of gas constant for dry air, Coriolis parameter, pressures of below and upper levels of atmospheric layer, unit vector of vertical axis of pressure coordinates system, specific humidity of air, air temperature and logarithm of air pressure.

Now, we consider equation (5-D-II-a), i.e.:

$$u_{\mathrm{D_{II}}} = \frac{R_d}{f}\int_{p_1}^{p_2}\frac{\partial T_{\mathrm{v}}}{\partial y}d\ln p \qquad \text{(5-D-II-a)}$$

First of all, we do integration on equation (5-D-II-a):

$$u_{\mathrm{D_{II}}} = \frac{R_d}{f}\frac{\partial\langle T_{\mathrm{v}}\rangle}{\partial y}\ln\left(\frac{p_2}{p_1}\right) \tag{8}$$

where in equation (8), $\langle \dots \rangle$ refers to vertical average of phrase or parameter in the layer enclosed by p_1 and p_2 pressure levels.[2]

If we select equivalent of T_v from equation (6), insert into equation (8) and applying derivation, yields:

$$u_{\mathrm{D_{II}}} = \frac{R_d}{f}\left[\left(\begin{array}{c} 0.608\frac{\partial\langle q\rangle}{\partial y}\langle T\rangle + \\ (1+0.608\langle q\rangle)\frac{\partial\langle T\rangle}{\partial y}\end{array}\right)\right]\ln\left(\frac{p_2}{p_1}\right) \quad \text{(9-D-II-a)}$$

And with the same manner, one can find out from equation (5-D-II-b):

$$v_{\mathrm{D_{II}}} = -\frac{R_d}{f}\left[\left(\begin{array}{c} 0.608\frac{\partial\langle q\rangle}{\partial x}\langle T\rangle + \\ (1+0.608\langle q\rangle)\frac{\partial\langle T\rangle}{\partial x}\end{array}\right)\right]\ln\left(\frac{p_2}{p_1}\right) \quad \text{(9-D-II-b)}$$

Equations (9-D-II-a) and (9-D-II-b) show component equations of second version of dense wind, in terms of gas constant for dry air, Coriolis parameter, specific humidity, temperature and pressure. In addition; symbol $\langle \dots \rangle$ refers to vertical average of phrase or parameter in the layer enclosed by p_1 and p_2 pressure levels.

And, as we noticed in part I; our aim in this research is, looking for geostrophic wind shear in the condition of real atmosphere.

However, a question that arises from the above-mentioned paragraph is; if we consider atmosphere as natural atmosphere including humidity; then how the looking to variation of the wind in vertical direction should be modify? In this research, the variation of the geostrophic wind with respect to height will be considered in the natural atmosphere, as well as dry air.

2.1. First special case of dense wind: Thermal wind

If the air would be assumed dry[3], i.e.:

$$q = 0 \tag{10}$$

or air is humid but specific humidity is constant[4] that is:

$$q = constant \tag{11}$$

or horizontal gradient of vertical mean for specific humidity in the atmospheric layer may be zero[5], so that:

$$\left[\frac{\partial\langle q\rangle}{\partial x} = 0 \ \text{and} \ \frac{\partial\langle q\rangle}{\partial y} = 0\right] \ \text{but} \ \frac{\partial q}{\partial p} \neq 0 \tag{12}$$

In these conditions; the variation of density in the horizontal direction is merely related to variation of temperature in horizontal direction causing baroclinity[6] of the atmosphere. In these circumstances; we define the vectorial difference of geostrophic wind vector at upper level and geostrophic wind vector at lower level of the atmospheric layer as, *thermal wind*, i.e.:

$$\mathbb{V}_{\mathrm{T}} \equiv \mathbb{V}_{g(p_2)} - \mathbb{V}_{g(p_1)} \quad \text{(1-T)}$$

where in Equation (1-T) $\mathbb{V}_{\mathrm{T}}$ stands for thermal wind vector, $\mathbb{V}_g$ is geostrophic wind and subscripts p_1 and p_2 refer to pressure levels in the manner that level p_2 has more height than level p_1, i.e., $p_2 < p_1$.

According to definition (1-T); eastward and northward components of thermal wind can be shown as following:

$$u_{\mathrm{T}} = u_{g(p_2)} - u_{g(p_1)} \quad \text{(2-T-a)}$$

and

$$v_{\mathrm{T}} = v_{g(p_2)} - v_{g(p_1)} \quad \text{(2-T-b)}$$

where in equations (2-T-a) and (2-T-b) u_{T} is eastward component of thermal wind, v_{T} is northward component of thermal wind, u_g is eastward component of geostrophic wind, v_g is northward component of geostrophic wind and subscripts p_1 and p_2 refer to pressure levels in the manner that level p_2 has more height than level p_1, i.e., $p_2 < p_1$.

Approaching to derive thermal wind has two branches. 1 – Temperature branch and 2 – Geopotential branch.

2.1.1 Thermal wind; Approach by Temperature

Approaching thermal wind by temperature has three particular types those will describe in following parts.

2.1.1.1 First particular type of the thermal wind

Now, we assume that air is dry by taking $q = 0$.

Geostrophic wind can be introduced by:

$$\mathbb{V}_g = f^{-1}\mathbb{k}_p \times \nabla_p \Phi \tag{13}$$

---

[2] For more details of "*Vertical averaging*" see [1]

[3] This case is not real, and assumption is for simplicity of deriving formulae of special cases of dense wind.

[4] Existence of this condition is possible for air over ocean especially at nights.

[5] This case can be occurred in tropic regions in the dry season. Also; it is possible to be the case at the surface of oceans in mid-latitude in days, upon conditions that: a) low difference between two latitudes and b) no existence of atmospheric system. However, this idea should be tested with observations.

[6] Baroclinity is the state of baroclinic atmosphere or baroclinic ocean. That state is existence of horizontal variation of density and is described in part I of "*A New Look at the Vertical Shear of the Geostrophic Wind*"

where in equation (13) $\mathbb{v}_g$ is geostrophic wind vector, f is Coriolis parameter, $\mathbb{k}$ is vertical unit vector, subscript p shows that equation (13) is written in pressure coordinates system, ∇ is operator for gradient, Φ is geopotential and again, subscript p shows that coordinates system is pressure coordinates system.
Furthermore, equation (13) shows the magnitude of geostrophic wind is proportional to the horizontal gradient of geopotential and is parallel to equipotential lines on isobaric surface. [10]
Writing eastward and northward components of geostrophic wind yields following equations:

$$u_g = -\frac{1}{f}\frac{\partial \Phi}{\partial y} \tag{14-a}$$

and

$$v_g = \frac{1}{f}\frac{\partial \Phi}{\partial x} \tag{14-b}$$

where in equation (14-a) u_g is eastward component of geostrophic wind or eastward component of geostrophic current in ocean, and in equation (14-b) v_g is northward component of geostrophic wind or northward component of geostrophic current in ocean.
Now, if we differentiate equations (14-a) and (14-b) with respect to p, we get:

$$\frac{\partial u_g}{\partial p} = -\frac{1}{f}\frac{\partial}{\partial p}\left(\frac{\partial \Phi}{\partial y}\right) = -\frac{1}{f}\frac{\partial}{\partial y}\left(\frac{\partial \Phi}{\partial p}\right) \tag{15-a}$$

and

$$\frac{\partial v_g}{\partial p} = \frac{1}{f}\frac{\partial}{\partial p}\left(\frac{\partial \Phi}{\partial x}\right) = \frac{1}{f}\frac{\partial}{\partial x}\left(\frac{\partial \Phi}{\partial p}\right) \tag{15-b}$$

The equation of state for dry air is: [10]

$$p = \rho R_d T \tag{16}$$

where in equation (16), p is pressure, ρ is dry air density, R_d is gas constant for dry air and T is its temperature. In pressure coordinates system; hydrostatic equation, applying for dry air and considering the equation of state for dry air, i.e., equation (16); is [13]:

$$\frac{\partial \Phi}{\partial p} = -\alpha = -\frac{1}{\rho} = -\frac{R_d T}{p} \tag{17}$$

where α is specific volume of dry air.
If we select equivalent of $\frac{\partial \Phi}{\partial p}$ from equation (17) that is $-\frac{R_d T}{p}$ and substitute in equations (15-a) and (15-b), we get:

$$\frac{\partial u_g}{\partial p} = -\frac{1}{f}\frac{\partial}{\partial y}\left(-\frac{R_d T}{p}\right) \tag{18-a}$$

and

$$\frac{\partial v_g}{\partial p} = \frac{1}{f}\frac{\partial}{\partial x}\left(-\frac{R_d T}{p}\right) \tag{18-b}$$

Rearranging equations (18-a) and (18-b) will be:

$$p\,\frac{\partial u_g}{\partial p} \equiv \frac{\partial u_g}{\partial \ln p} = \frac{R_d}{f}\left(\frac{\partial T}{\partial y}\right)_p \tag{19-T-I-a}$$

and

$$p\,\frac{\partial v_g}{\partial p} \equiv \frac{\partial v_g}{\partial \ln p} = -\frac{R_d}{f}\left(\frac{\partial T}{\partial x}\right)_p \tag{19-T-I-b}$$

where in equations (19-T-I-a) and (19-T-I-b) subscript p points that derivative is down with constant p. If we compound equations (19-T-I-a) and (19-T-I-b), we get "*Thermal wind equation by first particular type"* that is:

$$\frac{\partial \mathbb{v}_g}{\partial \ln p} = -\frac{R_d}{f}\left(\mathbb{k}_p \times \nabla_p T\right) \tag{3-T-I}$$

By integrating of equation (3-T-I) from lower pressure level p_1 to upper pressure level p_2 $(p_2 < p_1)$ of the atmospheric layer; one can derive "*First particular type of thermal wind vector*":

$$\mathbb{v}_{g(p_2)} - \mathbb{v}_{g(p_1)} \equiv \mathbb{v}_{\mathrm{T_I}} = -\frac{R_d}{f}\int_{p_1}^{p_2}\left(\mathbb{k}_p \times \nabla_p T\right) d \ln p \tag{4-T-I}$$

where in equation (4-T-I); $\mathbb{v}_{g(p_2)}$ is geostrophic wind vector at upper level of the atmospheric layer, $\mathbb{v}_{g(p_1)}$ is geostrophic wind vector at lower level of the atmospheric layer, $\mathbb{v}_{\mathrm{T_I}}$ stands for first particular type of thermal wind vector, R_d is gas constant for dry air, f is Coriolis parameter, p_1 is atmospheric pressure at lower level of the atmospheric layer, p_2 is atmospheric pressure at upper level of the atmospheric layer, $\mathbb{k}_p$ is vertical unit vector in pressure coordinates system, ∇_p stands for gradient operator in pressure coordinates system, T is dry air temperature and p is atmospheric pressure and vertical axis of pressure coordinates system.
Eastward and northward components of the first particular type of thermal wind can be derived by vertical integration of equations (19-T-I-a) and (19-T-I-b) in vertical coordinate same as integration of equation (3-T-I), or determine the eastward and northward components of the first particular type of thermal wind from equation (4-T-I) directly:

$$u_{\mathrm{T_I}} = \frac{R_d}{f}\int_{p_1}^{p_2}\frac{\partial T}{\partial y}\, d \ln p \tag{5-T-I-a}$$

and

$$v_{\mathrm{T_I}} = -\frac{R_d}{f}\int_{p_1}^{p_2}\frac{\partial T}{\partial x}\, d \ln p \tag{5-T-I-b}$$

where in equation (5-T-I-a), $u_{\mathrm{T_I}}$ is eastward component of first particular type of thermal wind and in equation (5-T-I-b), $v_{\mathrm{T_I}}$ is northward component of first particular type of thermal wind.
From first particular type of thermal wind vector or its components; one can find out that:

A: If we go from pole to equator, thermal wind becomes stronger[7];

B: If the horizontal gradient of temperature would be greater, thermal wind becomes stronger, because thermal wind is proportional to the horizontal gradient of temperature, and finally;

C: If the pressure difference will be higher in the layer, thermal wind becomes more powerful.

In this manner; representative of thermal wind vector, i.e., equation (4-T-I) shows that: "*thermal wind blows parallel to isotherms, so that, warm dry air is located at the right side of downwind and cold dry air is located at the left side of downwind.*" (In the northern hemisphere) This fact is illustrated in figures 3 and 4.

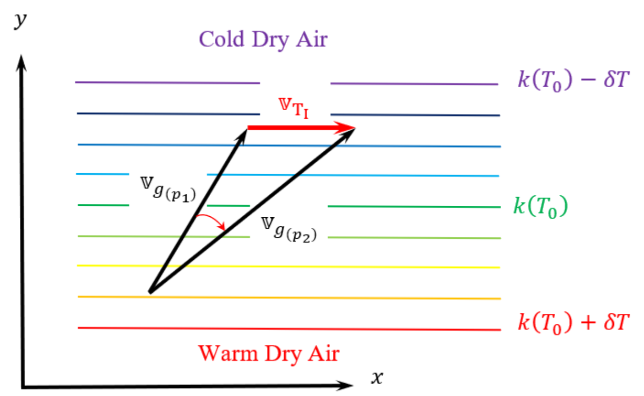

Figure 3. Relationship between clockwise turning of the geostrophic wind with respect to height (veering) and warm dry air advection. In the case; T_0 is mean temperature of the layer and proportion coefficient k is: $k = -\frac{R_d}{f}\ln\left(\frac{p_2}{p_1}\right)$

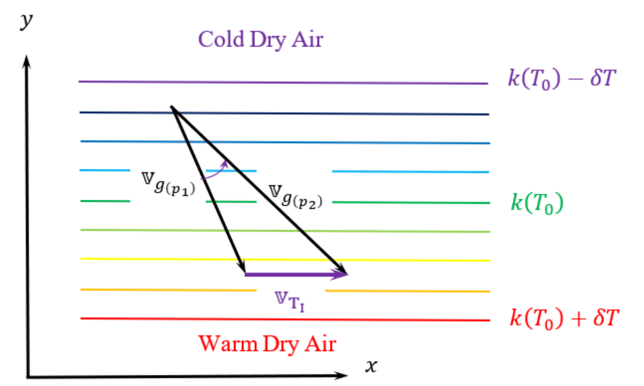

Figure 4. Relationship between counterclockwise turning of the geostrophic wind with respect to height (backing) and cold dry air advection. In the case; T_0 is mean temperature of the layer and proportion coefficient k is: $k = -\frac{R_d}{f}\ln\left(\frac{p_2}{p_1}\right)$

As it is shown in figure 3; in the dry atmosphere, in which specific humidity is zero; a geostrophic wind that turns clockwise with respect to height (veering) is associated with warm dry air advection. Conversely, as shown in figure 4; in the above-mentioned air, counterclockwise turning (backing) of the geostrophic wind with respect to height, implies cold dry air advection by the geostrophic wind in the layer.

It is therefore possible to obtain a reasonable estimate of the horizontal temperature advection and its vertical dependence at a given location solely from data on the vertical profile of the wind given by a single sounding. Alternatively, the geostrophic wind at any level can be estimated from the mean temperature field provided that the geostrophic velocity is known at a single level. Thus, for example, if the geostrophic wind at 850 hPa is known and the mean horizontal temperature gradient in the layer 850 – 500 hPa is also known, the first particular type of thermal wind equation can be applied to obtain the geostrophic wind at 500 hPa [10]

First particular type of thermal wind vector, i.e., equation (4-T-I) has a simple form by integration with respect to vertical axis, as following:

$$\mathbb{v}_{T_I} = -\frac{R_d}{f}\langle \mathbb{k}_p \times \nabla_p T\rangle \ln\left(\frac{p_2}{p_1}\right) \qquad (20\text{-T-I})$$

Also, equation (20-T-I) can be written in simpler form as:

$$\mathbb{v}_{T_I} = -\frac{R_d}{f}\left(\mathbb{k}_p \times \nabla_p\langle T\rangle\right) \ln\left(\frac{p_2}{p_1}\right) \qquad (21\text{-T-I})$$

In equations (20-T-I) and (21-T-I), $\langle \dots \rangle$ is vertical average of phrase or parameter.

Analogous to integration of equation (4-T-I); we can obtain simple form of equations (5-T-I-a) and (5-T-I-b) those show simple forms for components of first particular type of thermal wind vector for dry air; those are:

$$u_{T_I} = \frac{R_d}{f}\frac{\partial\langle T\rangle}{\partial y}\ln\left(\frac{p_2}{p_1}\right) \qquad (22\text{-T-I-a})$$

and

$$v_{T_I} = -\frac{R_d}{f}\frac{\partial\langle T\rangle}{\partial x}\ln\left(\frac{p_2}{p_1}\right) \qquad (22\text{-T-I-b})$$

Again, in equations (22-T-I-a) and (22-T-I-b), $\langle \dots \rangle$ is vertical averaging of phrase or parameter and furthermore, in equation (22-T-I-a) u_{T_I} is eastward component of thermal wind by first particular type and in equation (22-T-I-b) v_{T_I} is northward component of first particular type of thermal wind.

There is another method to derive first particular type of thermal wind; that at the moment, we don't refer to it.

2.1.1.2 Second particular type of the thermal wind

In this condition that air is humid but humidity of atmosphere is constant, i.e., $q = constant$; and of course, the variation of density in the horizontal direction is merely related to variation of temperature

[7] Use of geostrophic wind in tropical regions must be with careful deliberation because geostrophic wind in these regions is magnified and especially on equator is meaningless.

in horizontal direction and causes to produce baroclinic atmosphere. In the case, we define the vectorial difference of geostrophic wind vector at upper level and geostrophic wind vector at lower level of the atmospheric layer as, *thermal wind* again, i.e.:

$$\mathbb{V}_{\mathrm{T}} \equiv \mathbb{V}_{g(p_2)} - \mathbb{V}_{g(p_1)} \quad \text{(1-T)}$$

And according to definition (1-T); eastward and northward components of thermal wind can be shown as following:

$$u_{\mathrm{T}} = u_{g(p_2)} - u_{g(p_1)} \quad \text{(2-T-a)}$$

and

$$v_{\mathrm{T}} = v_{g(p_2)} - v_{g(p_1)} \quad \text{(2-T-b)}$$

The equation of state for humid or moist air can be written as: [1]

$$p = \rho_{\mathrm{M}} R_d T_v \quad (23)$$

Where p is pressure, ρ_{M} is density of humid air, R_d is gas constant for dry air and T_v is virtual temperature.
In pressure coordinates system, hydrostatic equation, applying for humid air and considering the equation of state; is [13]:

$$\frac{\partial \Phi}{\partial p} = -\alpha_{\mathrm{M}} = -\frac{R_d T_v}{p} \quad (24)$$

where in equation (24), Φ is geopotential, p is pressure as vertical coordinate, α_{M} is specific volume of humid air, R_d is constant gas for dry air and T_v is virtual temperature.
If we select equivalent of $\frac{\partial \Phi}{\partial p}$ from equation (24) that is $-\frac{R_d T_v}{p}$ and substitute in equations (15-a) and (15-b), we get:

$$\frac{\partial u_g}{\partial p} = -\frac{1}{f}\frac{\partial}{\partial y}\left(-\frac{R_d T_v}{p}\right) \quad \text{(25-a)}$$

and

$$\frac{\partial v_g}{\partial p} = \frac{1}{f}\frac{\partial}{\partial x}\left(-\frac{R_d T_v}{p}\right) \quad \text{(25-b)}$$

or

$$p\frac{\partial u_g}{\partial p} = \frac{R_d}{f}\left(\frac{\partial T_v}{\partial y}\right)_p \quad \text{(26-a)}$$

and

$$p\frac{\partial v_g}{\partial p} = -\frac{R_d}{f}\left(\frac{\partial T_v}{\partial x}\right)_p \quad \text{(26-b)}$$

where in equations (26-a) and (26-b) subscript p points that derivative is down with constant p.
Considering equations (26-a) and (26-b) and substitute equivalent of T_v form equation (6) into these equations yields:

$$p\frac{\partial u_g}{\partial p} = \frac{R_d}{f}\left[\frac{\partial(1+0.608q)T}{\partial y}\right]_p \quad \text{(27-a)}$$

and

$$p\frac{\partial v_g}{\partial p} = -\frac{R_d}{f}\left[\frac{\partial(1+0.608q)T}{\partial x}\right]_p \quad \text{(27-b)}$$

or

$$\frac{\partial u_g}{\partial \ln p} = \frac{R_d}{f}\left[\frac{\partial(1+0.608q)T}{\partial y}\right]_p \quad \text{(28-a)}$$

and

$$\frac{\partial v_g}{\partial \ln p} = -\frac{R_d}{f}\left[\frac{\partial(1+0.608q)T}{\partial x}\right]_p \quad \text{(28-b)}$$

By considering condition (11), i.e., $q = constant$, we can rewrite equations (28-a) and (28-b) as following:

$$\frac{\partial u_g}{\partial \ln p} = (1+0.608q)\frac{R_d}{f}\left(\frac{\partial T}{\partial y}\right)_p \quad \text{(19-T-II-a)}$$

and

$$\frac{\partial v_g}{\partial \ln p} = -(1+0.608q)\frac{R_d}{f}\left(\frac{\partial T}{\partial x}\right)_p \quad \text{(19-T-II-b)}$$

Equations (19-T-II-a) and (19-T-II-b) can be combined as vector form:

$$\frac{\partial \mathbb{V}_g}{\partial \ln p} = -(1+0.608q)\frac{R_d}{f}\left(\mathbb{k}_p \times \nabla_p T\right) \quad \text{(3-T-II)}$$

Equation (3-T-II) is "*Second particular type of thermal wind equation*".
By integration of equation (3-T-II) from lower pressure level p_1 to upper pressure level p_2, $(p_2 < p_1)$ of the atmospheric layer; one can derive "*Second particular type of thermal wind vector*" that is:

$$\mathbb{V}_{g(p_2)} - \mathbb{V}_{g(p_1)} \equiv \mathbb{V}_{\mathrm{T_{II}}} = -(1+0.608q)\frac{R_d}{f}\int_{p_1}^{p_2}\left(\mathbb{k}_p \times \nabla_p T\right) d\ln p \quad \text{(4-T-II)}$$

where in equation (4-T-II); $\mathbb{V}_{g(p_2)}$ is geostrophic wind vector at upper level of the atmospheric layer, $\mathbb{V}_{g(p_1)}$ is geostrophic wind vector at lower level of the atmospheric layer with constant specific humidity, $\mathbb{V}_{\mathrm{T_{II}}}$ stands for second particular type of thermal wind vector, q is specific humidity, R_d is gas constant for dry air, f is Coriolis parameter, p_1 is atmospheric pressure at lower level of the atmospheric layer, p_2 is atmospheric pressure at upper level of the atmospheric layer, $\mathbb{k}_p$ is vertical unit vector in pressure coordinates system, ∇_p stands for gradient operator in pressure coordinates system, T is air temperature and p is atmospheric pressure and vertical axis of pressure coordinates system.
Eastward and northward components of second particular type of thermal wind can be derived by

vertical integration of equations (30-T-II-a) and (30-T-II-b) same as integration of equation (3-T-II), or determine the eastward and northward components of second particular type of thermal wind from equation (4-T-II) directly:

$$\int_{p_1}^{p_2} du_g = (1+0.608q)\frac{R_d}{f}\int_{p_1}^{p_2}\left(\frac{\partial T}{\partial y}\right) d\ln p \qquad (29\text{-a})$$

and

$$\int_{p_1}^{p_2} dv_g = -(1+0.608q)\frac{R_d}{f}\int_{p_1}^{p_2}\frac{\partial T}{\partial x} d\ln p \qquad (29\text{-b})$$

And after doing integration to left hand on both above mentioned equations, we get:

$$u_{g(p_2)} - u_{g(p_1)} \equiv$$
$$u_{T_{II}} = (1+0.608q)\frac{R_d}{f}\int_{p_1}^{p_2}\left(\frac{\partial T}{\partial y}\right) d\ln p \qquad (5\text{-T-II-a})$$

and

$$v_{g(p_2)} - v_{g(p_1)} \equiv$$
$$v_{T_{II}} = -(1+0.608q)\frac{R_d}{f}\int_{p_1}^{p_2}\left(\frac{\partial T}{\partial x}\right) d\ln p \qquad (5\text{-T-II-b})$$

where in equation (5-T-II-a), $u_{T_{II}}$ is eastward component of the second particular type of thermal wind, and in equation (5-T-II-b), $v_{T_{II}}$ is northward component of the second particular type of thermal wind.

From second particular type of thermal wind vector or its components; one can find out that:

A: If we go from pole to equator, thermal wind becomes stronger (with pay attention to footnote No. 7);

B: If the horizontal gradient of temperature would be greater, thermal wind becomes stronger, because thermal wind is proportional to the horizontal gradient of temperature;

C: If the pressure difference will be higher in the layer, thermal wind becomes more powerful, and finally;

D: If humid air has more specific humidity, then thermal wind will be stronger.

In this manner; representative of second particular type of thermal wind vector, i.e., equation (4-T-II) shows that: "*thermal wind blows parallel to isotherms, so that, warm humid air is located at the right side of downwind and cold humid air is located at the left side of downwind.*" (In the northern hemisphere) This fact is illustrated in figures 5 and 6.

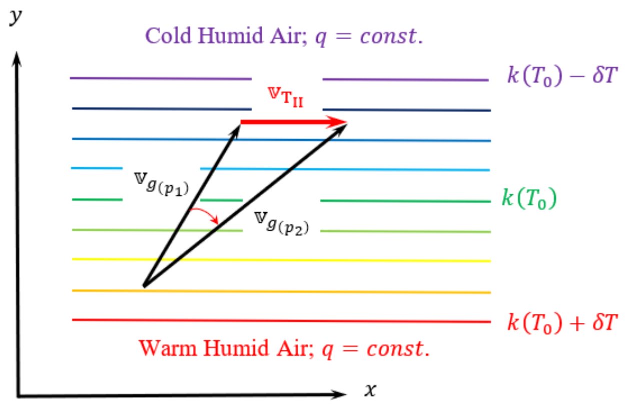

Figure 5. Relationship between clockwise turning of the geostrophic wind with respect to height (veering) and warm humid air advection. In the case; T_0 is mean temperature of the layer and proportion coefficient k is:

$$\boldsymbol{k = -(1+0.608q)\frac{R_d}{f}\ln\left(\frac{p_2}{p_1}\right)}$$

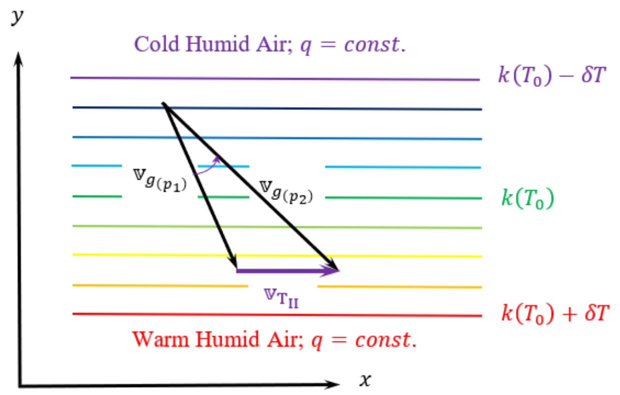

Figure 6. Relationship between counterclockwise turning of the geostrophic wind with respect to height (backing) and cold humid air advection. In the case; T_0 is mean temperature of the layer and proportion coefficient k is:

$$\boldsymbol{k = -(1+0.608q)\frac{R_d}{f}\ln\left(\frac{p_2}{p_1}\right)}$$

As it is shown in figure 5; in the humid atmosphere with constant specific humidity; a geostrophic wind that turns clockwise with respect to height (veering) is associated with warm humid air advection. Conversely, as shown in figure 6; in the above-mentioned air, counterclockwise turning (backing) of the geostrophic wind with respect to height, implies cold humid air advection by the geostrophic wind in the atmospheric layer.

Moreover, for second particular type of thermal wind; conditions those were mentioned under first particular type of thermal wind for obtaining horizontal warm humid air advection, cold humid air advection, mean temperature field or geostrophic wind at one level of the atmospheric layer with humid air, by knowing other parameters are true.

Second particular type of thermal wind vector, i.e., equation (4-T-II) has a simple form by integration with respect to vertical axis, as following:

$$\mathbb{v}_{\mathrm{T_{II}}} = -(1+0.608q)\frac{R_d}{f}\langle \mathbb{k}_p \times \nabla_p T\rangle \ln\left(\frac{p_2}{p_1}\right) \qquad \text{(20-T-II)}$$

Also, equation (20--T-II) can be written in simpler form as:

$$\mathbb{v}_{\mathrm{T_{II}}} = -(1+0.608q)\frac{R_d}{f}\left(\mathbb{k}_p \times \nabla_p \langle T\rangle\right) \ln\left(\frac{p_2}{p_1}\right) \qquad \text{(21-T-II)}$$

where in equations (20-T-II) and (21-T-II), $\langle \ldots \rangle$ is vertical averaging of phrase or parameter.
Analogous to integration of equation (4-T-II); we can obtain simple form of equations (5-T-II-a) and (5-T-II-b) those show simple forms for components of second particular type of thermal wind vector for humid air with constant specific humidity; those are:

$$u_{\mathrm{T_{II}}} = (1+0.608q)\frac{R_d}{f}\frac{\partial\langle T\rangle}{\partial y}\ln\left(\frac{p_2}{p_1}\right) \qquad \text{(22-T-II-a)}$$

and

$$v_{\mathrm{T_{II}}} = -(1+0.608q)\frac{R_d}{f}\frac{\partial\langle T\rangle}{\partial x}\ln\left(\frac{p_2}{p_1}\right) \qquad \text{(22-T-II-b)}$$

Again, in equations (22-T-II-a) and (22-T-II-b), $\langle \ldots \rangle$ is vertical averaging of phrase or parameter and furthermore, in equation (22-T-II-a) $u_{\mathrm{T_{II}}}$ is eastward component of thermal wind by second particular type and in equation (22-T-II-b) $v_{\mathrm{T_{II}}}$ is northward component of thermal wind in second particular type.
There is another method to derive second particular type of thermal wind; that at the moment, we don't refer to it.

2.1.1.3 Third particular type of the thermal wind

In this case; air has humidity but we assume that horizontal gradient of vertical average of specific humidity in the atmospheric layer may be zero but specific humidity varies in vertical direction, so that:

$$\left[\frac{\partial\langle q\rangle}{\partial x} = 0 \ \text{ and } \ \frac{\partial\langle q\rangle}{\partial y} = 0\right] \ \text{ but } \ \frac{\partial q}{\partial p} \neq 0 \qquad (12)$$

And in this case, atmosphere medium is very similar to barotropic medium, where in barotropic medium, we have not variation of density in horizontal direction, but density can vary in vertical direction.
In this circumstance; the variation of density in the horizontal direction is only related to variation of temperature in horizontal direction and causes to produce baroclinic atmosphere. In the situation, we define the vectorial difference of geostrophic wind vector at upper level and geostrophic wind vector at lower level of the atmospheric layer as, *thermal wind* again, i.e.:

$$\mathbb{v}_{\mathrm{T}} = \mathbb{v}_{g(p_2)} - \mathbb{v}_{g(p_1)} \qquad \text{(1-T)}$$

And according to definition (1-T); eastward and northward components of thermal wind can be shown as following:

$$u_{\mathrm{T}} = u_{g(p_2)} - u_{g(p_1)} \qquad \text{(2-T-a)}$$

and

$$v_{\mathrm{T}} = v_{g(p_2)} - v_{g(p_1)} \qquad \text{(2-T-b)}$$

For deriving all equations related to this case; firstly, we consider equations (5-D-II-a) and (5-D-II-b), i.e.:

$$u_{\mathrm{D_{II}}} = \frac{R_d}{f}\int_{p_1}^{p_2}\frac{\partial T_{\mathrm{v}}}{\partial y}\, d\ln p \qquad \text{(5-D-II-a)}$$

and

$$v_{\mathrm{D_{II}}} = -\frac{R_d}{f}\int_{p_1}^{p_2}\frac{\partial T_{\mathrm{v}}}{\partial x}\, d\ln p \qquad \text{(5-D-II-b)}$$

and by substituting equivalent of T_v form equation (6) into above-mentioned equations, we get:

$$u_{\mathrm{D_{II}}} = \frac{R_d}{f}\int_{p_1}^{p_2}\frac{\partial}{\partial y}[(1+0.608q)T]\, d\ln p \qquad \text{(30-D-II-a)}$$

and

$$v_{\mathrm{D_{II}}} = -\frac{R_d}{f}\int_{p_1}^{p_2}\frac{\partial}{\partial x}[(1+0.608q)T]\, d\ln p \qquad \text{(30-D-II-b)}$$

By applying derivation $\frac{\partial}{\partial y}$ into equation (30-D-II-a), we have:

$$u_{\mathrm{D_{II}}} = \frac{R_d}{f}\left\{\begin{array}{l}\int_{p_1}^{p_2}\left(0.0608\frac{\partial q}{\partial y}\right)T\, d\ln p \,+ \\ \int_{p_1}^{p_2}(1+0.608q)\frac{\partial T}{\partial y}\, d\ln p\end{array}\right\} \qquad (31)$$

Calculation of first integral of equation (31) yields:

$$\left(0.0608\frac{\partial\langle q\rangle}{\partial y}\langle T\rangle\right)\ln\left(\frac{p_2}{p_1}\right) \qquad (32)$$

And according to conditions (12); $\frac{\partial\langle q\rangle}{\partial y}$ is equal to zero, therefore the first integral of equation (31) vanishes and equation (31) for eastward component of dense wind, reduces to eastward component of third particular type of thermal wind, that is:

$$u_{\mathrm{T_{III}}} = \frac{R_d}{f}\int_{p_1}^{p_2}(1+0.608q)\frac{\partial T}{\partial y}\, d\ln p \qquad \text{(5-T-III-a)}$$

where in equation (5-T-III-a), $u_{\mathrm{T_{III}}}$ is eastward component of third particular type of thermal wind.
By applying derivation $\frac{\partial}{\partial x}$ into equation (30-D-II-b) and same mathematical manipulation on equation (30-D-II-a) into this equation, one can derive northward component of third particular type of thermal wind, i.e.

$$v_{\mathrm{T_{III}}} = -\frac{R_d}{f}\int_{p_1}^{p_2}(1+0.608q)\frac{\partial T}{\partial x}\, d\ln p \qquad \text{(5-T-III-b)}$$

where in equation (5-T-III-b), $v_{\mathrm{T_{III}}}$ is northward component of third particular type of thermal wind.
Equation (5-T-III-a) used definition of third particular type of thermal wind, i.e.:

$$u_{\mathrm{T_{III}}} \equiv u_{g(p_2)} - u_{g(p_1)} = \int_{p_1}^{p_2} du_g \qquad \text{(2-T-III-a)}$$

Substituting of equivalent $u_{\mathrm{T_{III}}}$, i.e., $\int_{p_1}^{p_2} du_g$ from equation (2-T-III-a) into equation (5-T-III-a) yields:

$$\int_{p_1}^{p_2} du_g = \frac{R_d}{f}\int_{p_1}^{p_2}(1+0.608q)\frac{\partial T}{\partial y}d\ln p \qquad (33)$$

Derivation and rearranging equation (33) have following result:

$$\frac{\partial u_g}{\partial \ln p} = \frac{R_d}{f}(1+0.608q)\frac{\partial T}{\partial y} \qquad \text{(19-T-III-a)}$$

By same mathematical manipulation on equation (5-T-III-b), one can get equation (19-T-III-b) for northward component of third particular thermal wind:

$$\frac{\partial v_g}{\partial \ln p} = -\frac{R_d}{f}(1+0.608q)\frac{\partial T}{\partial x} \qquad \text{(19-T-III-b)}$$

Combining equations (19-T-III-a) and (19-T-III-b) yields:

$$\frac{\partial \mathbb{v}_g}{\partial \ln p} = -\frac{R_d}{f}(1+0.608q)\left(\mathbb{k}_p \times \nabla_p T\right) \qquad \text{(3-T-III)}$$

Equation (3-T-III) is "*Third particular type of thermal wind equation*"
Equations (19-T-III-a), (19-T-III-b) and (3-T-III) seem to be similar to equations (19-T-II-a), (19-T-II-b) and (3-T-II) exactly, but there are different between specific humidity for first three above mentioned equations with others. In equations (19-T-II-a), (19-T-II-b) and (3-T-II) specific humidity, q is constant whereas in equations (19-T-III-a), (19-T-III-b) and (3-T-III); although vertical average of specific humidity is same in horizontal direction, i.e., $\frac{\partial\langle q\rangle}{\partial x} = 0$ and $\frac{\partial\langle q\rangle}{\partial y} = 0$; but, in recent equations, $\frac{\partial q}{\partial p} \neq 0$.
By integrating of equation (3-T-III) from lower pressure level p_1 to upper pressure level p_2, ($p_2 < p_1$) of the atmospheric layer; one can derive "*Third particular type of thermal wind vector*" that is:

$$\mathbb{v}_{\mathrm{T_{III}}} = \begin{matrix}\mathbb{v}_{g(p_2)} - \mathbb{v}_{g(p_1)} \equiv \\ -\frac{R_d}{f}\int_{p_1}^{p_2}(1+0.608q)\left(\mathbb{k}_p \times \nabla_p T\right)d\ln p\end{matrix} \qquad \text{(4-T-III)}$$

where in equation (4-T-III); $\mathbb{v}_{g(p_2)}$ is geostrophic wind vector at upper level of the humid atmospheric layer, $\mathbb{v}_{g(p_1)}$ is geostrophic wind vector at lower level of the humid atmospheric layer with following specifications:

$$\left[\frac{\partial\langle q\rangle}{\partial x} = 0 \text{ and } \frac{\partial\langle q\rangle}{\partial y} = 0\right] \text{ but } \frac{\partial q}{\partial p} \neq 0 \qquad (12)$$

and, $\mathbb{v}_{\mathrm{T_{III}}}$ stands for third particular type of thermal wind vector, R_d is gas constant for dry air, f is Coriolis parameter, p_1 is atmospheric pressure at lower level of the atmospheric layer, p_2 is atmospheric pressure at upper level of the atmospheric layer, q is specific humidity, $\mathbb{k}_p$ is vertical unit vector in pressure coordinates system, ∇_p stands for gradient operator in pressure coordinates system, T is air temperature and p is atmospheric pressure and vertical axis of pressure coordinates system.
Eastward and northward components of third particular type of thermal wind introduced in equations (5-T-III-a) and (5-T-II-b) earlier.
From third particular type of thermal wind vector or its components; one can find out that:
A: If we go from pole to equator, thermal wind becomes stronger (with pay attention to footnote No. 7);
B: If the horizontal gradient of temperature would be greater, thermal wind becomes stronger, because thermal wind is proportional to the horizontal gradient of temperature;
C: If the pressure difference will be higher in the atmospheric layer, thermal wind becomes more powerful, and finally;
D: If Atmospheric layer has more specific humidity and specific humidity difference will be higher from below level until upper level of the atmospheric layer; then thermal wind will be stronger.
In this manner; representative of third particular type of thermal wind vector, i.e., equation (4-T-III) with consideration of conditions (12) shows that: "*thermal wind blows parallel to isotherms, so that, warm humid air is located at the right side of downwind and cold humid air is located at the left side of downwind.*" (In the northern hemisphere) This fact is illustrated in figures 7 and 8.

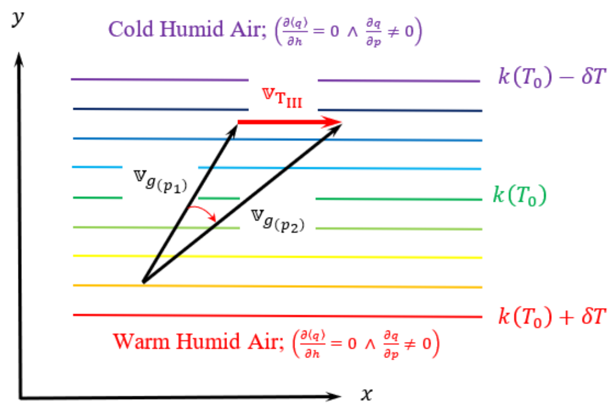

Figure 7. Relationship between clockwise turning of the geostrophic wind with respect to height (veering) and warm humid air advection. Conditions are $\left(\frac{\partial\langle q\rangle}{\partial x} = \frac{\partial\langle q\rangle}{\partial y} = 0 \equiv \frac{\partial\langle q\rangle}{\partial h} = 0 \wedge \frac{\partial q}{\partial p} \neq 0\right)$ and in the case; T_0 is mean temperature of the layer and proportion coefficient k is:
$$k = -\frac{R_d}{f}(1+0.608\langle q\rangle)\ln\left(\frac{p_2}{p_1}\right)$$

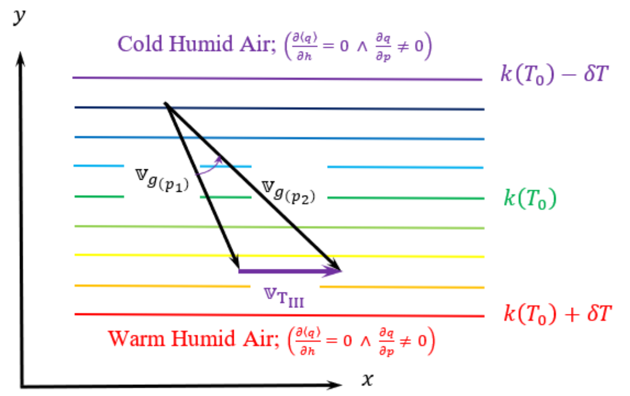

Figure 8. Relationship between counterclockwise turning of the geostrophic wind with respect to height (backing) and cold humid air advection. Conditions are $\left(\frac{\partial\langle q\rangle}{\partial x}=\frac{\partial\langle q\rangle}{\partial y}=0\equiv\frac{\partial\langle q\rangle}{\partial h}=0 \wedge \frac{\partial q}{\partial p}\neq 0\right)$ and in the case; T_0 is mean temperature of the layer and proportion coefficient k is:
$\boldsymbol{k=-\frac{R_d}{f}(1+0.608\langle q\rangle)\ln\left(\frac{p_2}{p_1}\right)}$

As it is shown in figure 7; in the humid atmosphere with consideration conditions (12); a geostrophic wind that turns clockwise with respect to height (veering) is associated with warm humid air advection. Conversely, as shown in figure 8; in the above-mentioned air, counterclockwise turning (backing) of the geostrophic wind with respect to height, implies cold humid air advection by the geostrophic wind in the layer.
Moreover, for third particular type of thermal wind; conditions those were mentioned under first particular type of thermal wind for obtaining cold humid air advection, warm humid air advection, mean temperature field or geostrophic wind at one level of the atmospheric layer with humid air considering conditions (12), by knowing other parameters are true.
Third particular type of thermal wind vector, i.e., equation (4-T-III) has a simple form when we bring out first parenthesis from integrand:

$$\mathbb{V}_{T_{III}}=-\frac{R_d}{f}(1+0.608\langle q\rangle)\int_{p_1}^{p_2}\left(\mathbb{k}_p\times\nabla_p T\right)d\ln p \tag{20-T-III}$$

and simpler form as following:

$$\mathbb{V}_{T_{III}}=-\frac{R_d}{f}(1+0.608\langle q\rangle)\left(\mathbb{k}_p\times\nabla_p\langle T\rangle\right)\ln\left(\frac{p_2}{p_1}\right) \tag{21-T-III}$$

Also, equations (5-T-III-a) and (5-T-III-b) have simple forms as follows:

$$u_{T_{III}}=\frac{R_d}{f}(1+0.608\langle q\rangle)\int_{p_1}^{p_2}\frac{\partial T}{\partial y}d\ln p \tag{34-T-III-a}$$

and

$$v_{T_{III}}=-\frac{R_d}{f}(1+0.608\langle q\rangle)\int_{p_1}^{p_2}\frac{\partial T}{\partial x}d\ln p \tag{34-T-III-b}$$

Even one can write simpler forms of equations (5-T-III-a) and (5-T-III-b) same as following equations:

$$u_{T_{III}}=\frac{R_d}{f}(1+0.608\langle q\rangle)\frac{\partial\langle T\rangle}{\partial y}\ln\left(\frac{p_2}{p_1}\right) \tag{22-T-III-a}$$

and

$$v_{T_{III}}=-\frac{R_d}{f}(1+0.608\langle q\rangle)\frac{\partial\langle T\rangle}{\partial x}\ln\left(\frac{p_2}{p_1}\right) \tag{22-T-III-b}$$

where in equations (20-T-III), (21-T-III), (34-T-III-a), (34-T-III-b), (22-T-III-a) and (22-T-III-b); $\langle\ldots\rangle$ refers to vertical average of phrase or parameter.
There is another method to derive third particular type of thermal wind; that at the moment, we don't refer to it.

2.1.2 Thermal wind; Approach by Geopotential

Now, we may express the thermal wind for a given atmospheric layer in terms of the geopotential or derivations of it, in the layer. In this case we assume; air is dry, has constant humidity or horizontal gradient of vertical average of specific humidity is zero. i.e.:

$$q=0 \tag{10}$$

or

$$q=constant \tag{11}$$

or

$$\left[\frac{\partial\langle q\rangle}{\partial x}=0 \text{ and } \frac{\partial\langle q\rangle}{\partial y}=0\right] \text{ but } \frac{\partial q}{\partial p}\neq 0 \tag{12}$$

We know, these conditions show that the variation of density in the horizontal direction is merely related to the variation of temperature in horizontal direction causing baroclinity of atmosphere, but we don't refer to these conditions in this section.
Also, approaching thermal wind by geopotential has two important types that we propound them.

2.1.2.1 Fourth particular type of the thermal wind

Geostrophic wind can be introduced by:

$$\mathbb{V}_g=f^{-1}\mathbb{k}_p\times\nabla_p\Phi \tag{13}$$

Writing eastward and northward components of geostrophic wind yields following equations:

$$u_g=-\frac{1}{f}\frac{\partial\Phi}{\partial y} \tag{14-a}$$

and

$$v_g=\frac{1}{f}\frac{\partial\Phi}{\partial x} \tag{14-b}$$

By vertical integrating of equation (14-a) we get:

$$\int_{p_1}^{p_2} u_g\, dp = -\frac{1}{f}\int_{p_1}^{p_2} \frac{\partial \Phi}{\partial y} dp \tag{35}$$

And by considering figure 2 we have:

$$u_{g(p_2)} - u_{g(p_1)} \equiv$$
$$u_{\mathrm{T_{IV}}} = -\frac{1}{f}\frac{\partial}{\partial y}(\Phi_2 - \Phi_1) \tag{5-T-IV-a}$$

where in equation (5-T-IV-a), $u_{\mathrm{T_{IV}}}$ is eastward component of fourth particular type of thermal wind.
By same mathematical manipulation related to equation (14-a) on equation (14-b), one can get:

$$v_{g(p_2)} - v_{g(p_1)} \equiv$$
$$v_{\mathrm{T_{IV}}} = \frac{1}{f}\frac{\partial}{\partial x}(\Phi_2 - \Phi_1) \tag{5-T-IV-b}$$

where in equation (5-T-IV-b), $v_{\mathrm{T_{IV}}}$ is northward component of fourth particular type of thermal wind.
Combining equations (5-T-IV-a) and (5-T-IV-b) yields:

$$\mathbb{v}_{g(p_2)} - \mathbb{v}_{g(p_1)} \equiv$$
$$\mathbb{v}_{\mathrm{T_{IV}}} = \frac{1}{f}\left[\mathbb{k}_p \times \nabla_p(\Phi_2 - \Phi_1)\right] \tag{4-T-IV}$$

where in equation (4-T-IV); $\mathbb{v}_{g(p_2)}$ is geostrophic wind vector at upper level of the atmospheric layer, $\mathbb{v}_{g(p_1)}$ is geostrophic wind vector at lower level of the atmospheric layer, $\mathbb{v}_{\mathrm{T_{IV}}}$ stands for *fourth particular type of thermal wind vector*, f is Coriolis parameter, $\mathbb{k}_p$ is vertical unit vector in pressure coordinates system, ∇_p stands for gradient operator in pressure coordinates system, Φ_2 is geopotential at upper level of the atmospheric layer and Φ_1 is geopotential at lower level of the atmospheric layer.
The term "geopotential height" refers to height of a given point in the atmosphere in units proportional to the potential energy of unit mass (geopotential) at this height relative to -mean- sea level. [10]
The relation, in SI units, between the geopotential height Z and geometric height z is:

$$Z = \frac{1}{g_0}\int_0^z g dz \tag{36}$$

where g is acceleration of gravity and g_0 is the globally averaged acceleration of gravity at sea level ($g_0 = 9.80665\ \mathrm{ms^{-2}}$) so that the two heights are numerically interchangeable for most meteorological purposes.
Therefore, for most purposes, it is sufficiently accurate to take gravitational acceleration g as a constant, given approximately by:

$$g \approx g_0 \approx 9.8\ \mathrm{ms^{-2}} \tag{37}$$

If the sea is at rest, its surface would coincide with the geopotential surface[8]. This geopotential surface is called -mean- sea level and is defined as $\Phi = 0$. To a good approximation, so the vertical coordinate z measures distance upward from this reference level, so

$$\Phi \approx gz \approx g_0 z \tag{38}$$

Geopotential is sometimes given in units of the geopotential meter (gpm) defined by:

$$1\ \mathrm{gpm} = 9.8\ \mathrm{m^2s^{-2}} \equiv 1\ \mathrm{Jkg^{-1}} \tag{39}$$

so that, the value of the geopotential in geopotential meters is close to the height in meters. Alternatively, the geopotential height Z is defined by:

$$Z = {}^{\Phi}/_{g_0} \tag{40}$$

so that the geopotential height in meters is numerically the same as the geopotential in geopotential meters. [8]
Equation (38) shows geopotential is proportional to height; then:

$$(\Phi_2 - \Phi_1) \approx g(z_2 - z_1) \tag{41}$$

where in equation (41), Φ_2 is geopotential of upper level of the atmospheric layer, Φ_1 stands for geopotential of lower level of the atmospheric layer, g is gravitational acceleration, z_2 is height of upper level of the atmospheric layer and $z_1$stands for height of lower level of the atmospheric layer. And $(z_2 - z_1)$ is the thickness of the layer. (See figure 2)
Now, from fourth particular type of thermal wind vector or its components; one can find out that:
A: If we go from pole to equator, fourth particular type of thermal wind becomes stronger (with pay attention to footnote No. 7);
B: If the horizontal gradient of thickness of the layer would be greater, fourth particular type of thermal wind becomes stronger, because the thermal wind is proportional to the horizontal gradient of thickness of the atmospheric layer and:
C: If the geopotential difference between lower level and upper level of the atmospheric layer will be higher, thermal wind becomes more powerful.
In this manner; representative of fourth particular type of thermal wind vector, i.e., equation (4-T-IV) with consideration of condition (10) shows that: "*fourth particular type of thermal wind blows parallel to isopleth of thickness, so that, more thickness of the atmospheric layer is located at the right side of downwind and less thickness of the atmospheric layer is located at the left side of downwind.*" (In the northern hemisphere) This fact is illustrated in figures 9 and 10.

[8] Or "*Base geopotential*"

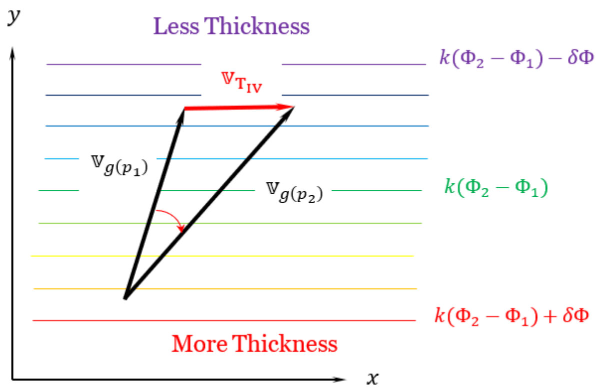

Figure 9. Relationship between clockwise turning of the geostrophic wind with respect to height (veering) and more thickness advection. In the case; proportion coefficient k is: $k = \frac{1}{f}$

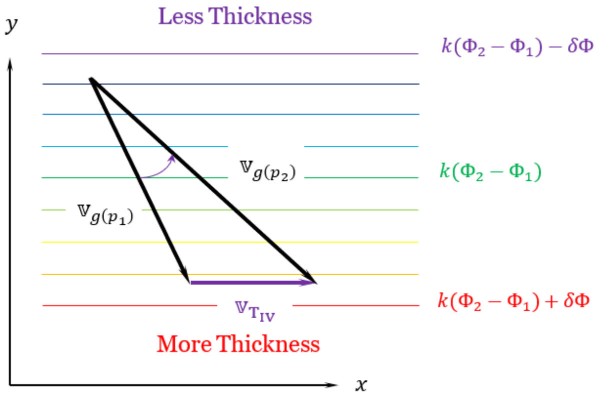

Figure 10. Relationship between counterclockwise turning of the geostrophic wind with respect to height (backing) and less thickness advection. In the case; proportion coefficient k is: $k = \frac{1}{f}$

As it is shown in figure 9; a geostrophic wind that turns clockwise with respect to height (veering) is associated with more thickness advection. Conversely, as shown in figure 10; counterclockwise turning (backing) of the geostrophic wind with respect to height, implies less thickness advection by the geostrophic wind in the layer.

It is therefore possible to obtain a reasonable estimate of the mean thickness change of the atmospheric layer between two near upper air stations from data on the vertical profile of the wind given by soundings at those upper air stations. Alternatively, the mean geostrophic wind at two points of one level of the atmospheric layer between two near upper air stations can be estimated from the mean thickness field advection of that layer provided that the mean geostrophic wind velocity is known at other level. Thus, for example, if the mean geostrophic wind velocity at 850 hPa is known from two near upper air stations and the mean gradient of the thickness in the layer 850 – 500 hPa between two near upper air stations is also known; the fourth particular type of thermal wind equation can be applied to obtain the mean geostrophic wind velocity in the layer at 500 hPa.

2.1.2.2 Fifth particular type of the thermal wind

Consider equations (5-T-IV-a) and (5-T-IV-b):

$$u_{T_{IV}} = -\frac{1}{f}\frac{\partial}{\partial y}(\Phi_2 - \Phi_1) \qquad \text{(5-T-IV-a)}$$

$$v_{T_{IV}} = \frac{1}{f}\frac{\partial}{\partial x}(\Phi_2 - \Phi_1) \qquad \text{(5-T-IV-b)}$$

If we replace equivalent of $(\Phi_2 - \Phi_1)$ from equation (41) into above-mentioned equations, we get:

$$u_{T_V} = -\frac{g}{f}\frac{\partial}{\partial y}(z_2 - z_1) \qquad \text{(5-T-V-a)}$$

$$v_{T_V} = \frac{g}{f}\frac{\partial}{\partial x}(z_2 - z_1) \qquad \text{(5-T-V-b)}$$

where in equation (5-T-V-a), u_{T_V} is eastward component of fifth particular type of thermal wind and in equation (5-T-V-b), $v_{T_V}$is northward component of fifth particular type of thermal wind and other parameters have been introduced under equation (41).

Combining equations (5-T-V-a) and (5-T-V-b) yields fifth particular type of thermal wind vector:

$$\mathbb{V}_{T_V} = \frac{g}{f}\left[\mathbb{k}_p \times \nabla_p[(z_2 - z_1)]\right] \qquad \text{(4-T-V)}$$

where in equation (4-T-V), $\mathbb{V}_{T_V}$ stands for *fifth particular type of thermal wind vector*, g is acceleration of gravity, f is Coriolis parameter, $\mathbb{k}_p$ is vertical unit vector in pressure coordinates system, ∇_p stands for gradient operator in pressure coordinates system, z_2 is altitude of upper level of the atmospheric layer and z_1 is altitude of lower level of the atmospheric layer.

From fifth particular type of thermal wind vector or its components; one can find out that:

A: If we go from pole to equator, fifth particular type of thermal wind becomes stronger (with pay attention to footnote No. 7);

B: If the horizontal gradient of thickness of the layer would be greater, fifth particular type of thermal wind becomes stronger, because the thermal wind is proportional to the horizontal gradient of thickness of the atmospheric layer and:

C: If the altitude difference between lower level and upper level of the atmospheric layer will be higher, thermal wind becomes more powerful.

In this manner; representative of fifth particular type of thermal wind vector, i.e., equation (4-T-V) shows that: "*fifth particular type of thermal wind blows parallel to isopleth of thickness, so that, more thickness of the atmospheric layer is located at the right side of downwind and less thickness of the atmospheric layer is located at the left side of downwind.*" (In the northern hemisphere) This fact is illustrated in figures 11 and 12.

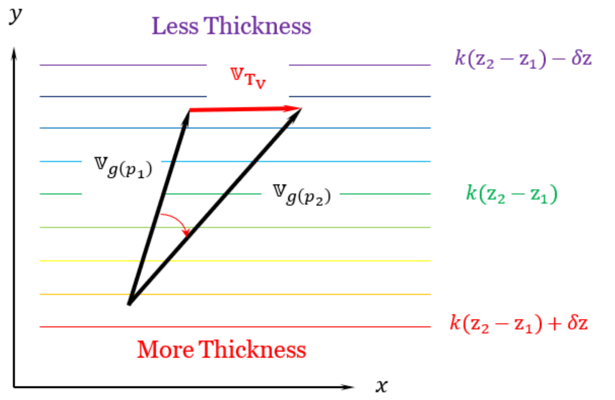

Figure 11. Relationship between clockwise turning of the geostrophic wind with respect to height (veering) and more thickness advection. In the case; proportion coefficient k is: $k = \frac{g}{f}$

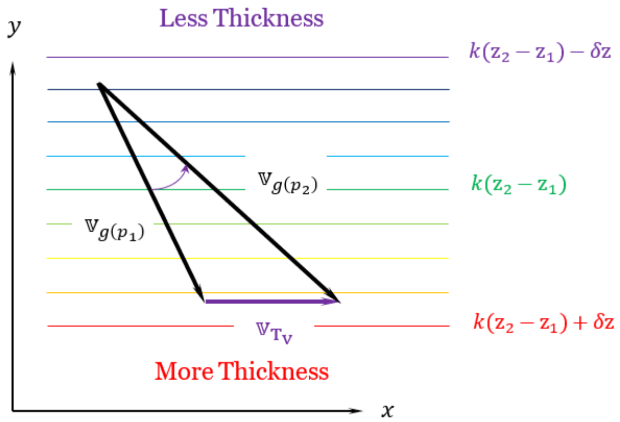

Figure 12. Relationship between counterclockwise turning of the geostrophic wind with respect to height (backing) and less thickness advection. In the case; proportion coefficient k is: $k = \frac{g}{f}$

As it is shown in figure 11; a geostrophic wind that turns clockwise with respect to height (veering) is associated with more thickness advection. Conversely, as shown in figure 12; counterclockwise turning (backing) of the geostrophic wind with respect to height, implies less thickness advection by the geostrophic wind in the layer.

Other specifications of fourth particular type of thermal wind are true for fifth particular type of thermal wind.

2.2. Second special case of dense wind: Moist wind

Another feature of dense wind that we introduce it as second special case of it, is moist wind.

If we assume that temperature in horizontal direction doesn't change;[9] – as it is true in most regions at tropics – i.e.

$$\frac{\partial T}{\partial x} = 0 \text{ and } \frac{\partial T}{\partial y} = 0 \tag{42}$$

In conditions No. (42); the variation of density in the horizontal direction is merely related to variation of humidity in horizontal direction causing baroclinity of the atmosphere. In these circumstances; we define the vectorial difference of geostrophic wind vector at upper level and geostrophic wind vector at lower level of the atmospheric layer as, *moist wind*, i.e.:

$$\mathbb{V}_{\mathrm{M}} \equiv \mathbb{V}_{g(p_2)} - \mathbb{V}_{g(p_1)} \tag{1-M}$$

where in Equation (1-M) $\mathbb{V}_{\mathrm{M}}$ stands for moist wind vector, $\mathbb{V}_g$ is geostrophic wind and subscripts p_1 and p_2 refer to pressure levels in the manner that level p_2 has more height than level p_1, i.e., $p_2 < p_1$.

According to definition (1-M); eastward and northward components of Moist wind can be shown as following:

$$u_{\mathrm{M}} = u_{g(p_2)} - u_{g(p_1)} \tag{2-M-a}$$

and

$$v_{\mathrm{M}} = v_{g(p_2)} - v_{g(p_1)} \tag{2-M-b}$$

where in equations (2-M-a) and (2-M-b) u_{M} is eastward component of moist wind, v_{M} is northward component of moist wind, u_g is eastward component of geostrophic wind, v_g is northward component of geostrophic wind and subscripts p_1 and p_2 refer to pressure levels in the manner that level p_2 has more height than level p_1, i.e., $p_2 < p_1$.

Also, approaching to derive moist wind equations has two branches. 1 – Temperature branch and 2 – Geopotential branch.

2.2.1 Moist wind; Approach by Temperature

For the second version of dense wind; dense wind equation was:

$$\frac{\partial \mathbb{V}_g}{\partial \ln p} = -\frac{R_d}{f}\left(\mathbb{k}_p \times \nabla_p T_v\right) \tag{3-D-II}$$

If we decompose equation (3-D-II) into its components, we get: [1]

$$\frac{\partial u_g}{\partial \ln p} = \frac{R_d}{f}\frac{\partial T_{\mathrm{v}}}{\partial y} \tag{43-a}$$

and

$$\frac{\partial v_g}{\partial \ln p} = -\frac{R_d}{f}\frac{\partial T_{\mathrm{v}}}{\partial x} \tag{43-b}$$

Substituting equivalent of T_v from equation (6) into equation (43-a) yields:

$$\frac{\partial u_g}{\partial \ln p} = \frac{R_d}{f}\frac{\partial(1+0.608q)T}{\partial y} \tag{44}$$

and applying derivative on right side of equation (44) yields:

[9] But can change in vertical direction i.e., $\frac{\partial T}{\partial p} \neq 0$

$$\frac{\partial u_g}{\partial \ln p} = \frac{R_d}{f}\left[\frac{\partial(1+0.608q)}{\partial y}T + (1+0.608q)\frac{\partial T}{\partial y}\right] \quad (45)$$

According to conditions (42) $\frac{\partial T}{\partial y} = 0$ and second term on the right hand of (45) vanishes. Now we have:

$$\frac{\partial u_g}{\partial \ln p} = (0.608)\frac{R_d T}{f}\left(\frac{\partial q}{\partial y}\right) \quad \text{(19-M-I-a)}$$

By same mathematical manipulation on equation (43-b), we have:

$$\frac{\partial v_g}{\partial \ln p} = -(0.608)\frac{R_d T}{f}\left(\frac{\partial q}{\partial x}\right) \quad \text{(19-M-I-b)}$$

Combining equations (19-M-I-a) and (19-M-I-b) yields:

$$\frac{\partial \mathbb{v}_g}{\partial \ln p} = -(0.608)\frac{R_d T}{f}\left(\mathbb{k}_p \times \nabla_p q\right) \quad \text{(3-M-I)}$$

Equation (3-M-I) is "*First prominent type of moist wind equation*"

By integrating of equation (3-M-I) from lower pressure level p_1 to upper pressure level p_2, $(p_2 < p_1)$ of the atmospheric layer (see figure 2); one can derive "*First prominent type of moist wind vector*" that is:

$$\mathbb{v}_{\mathrm{M_I}} \overset{\mathbb{v}_{g(p_2)} - \mathbb{v}_{g(p_1)} \equiv}{=} -(0.608)\frac{R_d}{f}\int_{p_1}^{p_2} T\left(\mathbb{k}_p \times \nabla_p q\right) d\ln p \quad \text{(4-M-I)}$$

where in equations (19-M-I-a), (19-M-I-b), (3-M-I) and (4-M-I): u_g is eastward component of geostrophic wind, v_g is northward component of geostrophic wind, $\mathbb{v}_g$ is geostrophic wind, $\mathbb{v}_{\mathrm{M_I}}$ stands for first prominent type of moist wind, R_d is gas constant for dry air, p_1 is lower pressure of the atmospheric layer, p_2 is upper pressure of the atmospheric layer, T is temperature, $\mathbb{k}_p$ is vertical unit vector in pressure coordinates system, q is specific humidity and p is pressure as vertical coordinate of pressure coordinates system.

Eastward and northward components of first prominent type of moist wind can derive from equation (4-M-I) easily. Those are in following lines:

$$u_{\mathrm{M_I}} = (0.608)\frac{R_d}{f}\int_{p_1}^{p_2} T\frac{\partial q}{\partial y}\, d\ln p \quad \text{(5-M-I-a)}$$

and:

$$v_{\mathrm{M_I}} = -(0.608)\frac{R_d}{f}\int_{p_1}^{p_2} T\frac{\partial q}{\partial x}\, d\ln p \quad \text{(5-M-I-b)}$$

where in equation (5-M-I-a), $u_{\mathrm{M_I}}$ is eastward component of first prominent type of the moist wind and in equation (5-M-I-b), $v_{\mathrm{M_I}}$ stands for northward component of first prominent type of the moist wind and other parameters were defined after equation (4-M-I).

From first prominent type of moist wind vector or its components; one can find out that:

A: If we go from pole to equator, moist wind becomes stronger (with pay attention to footnote No. 7);

B: Increasing temperature causes to produce power-up of moist wind;

C: If the horizontal gradient of specific humidity would be greater, moist wind becomes stronger, because moist wind is proportional to the horizontal gradient of specific humidity and finally;

D: If the pressure difference will be higher in the atmospheric layer, moist wind becomes more powerful.

In this manner; representative of first prominent type of moist wind vector, i.e., equation (4-M-I) with consideration of conditions (42) shows that: "*moist wind blows parallel to isopleths of specific humidity, so that, air with more specific humidity is located at the right side of downwind and air with less specific humidity is located at the left side of downwind.*" (In the northern hemisphere) This fact is illustrated in figures 13 and 14.

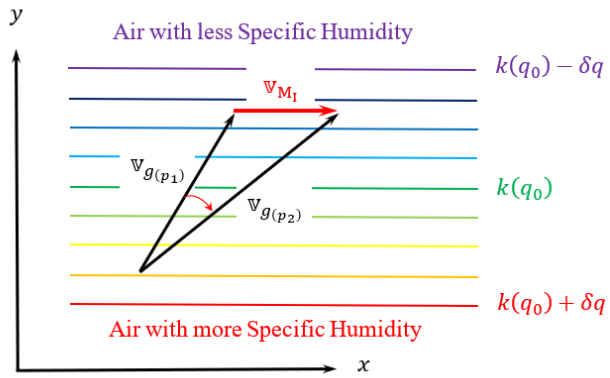

Figure 13. Relationship between clockwise turning of the geostrophic wind with respect to height (veering) and more specific humidity advection. Conditions: q_0 is mean specific humidity of the atmospheric layer and in the case; proportion coefficient k is: $k = -(0.608)\frac{R_d}{f}\langle T\rangle \ln\left(\frac{p_2}{p_1}\right)$

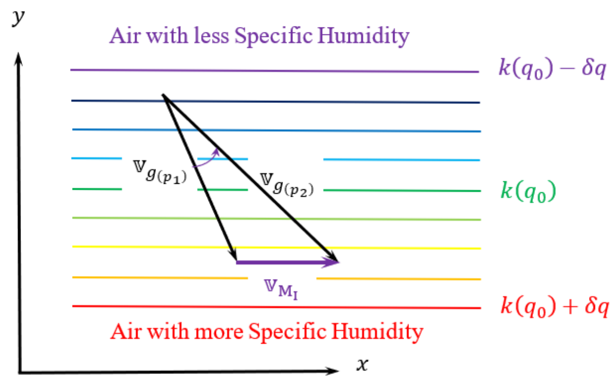

Figure 14. Relationship between counterclockwise turning of the geostrophic wind with respect to height (backing) and less specific humidity advection. Conditions: q_0 is mean specific humidity of the atmospheric layer and in the case; proportion coefficient k is: $k = -(0.608)\frac{R_d}{f}\langle T\rangle \ln\left(\frac{p_2}{p_1}\right)$

As it is shown in figure 13; in the humid atmosphere with consideration condition (42) – in the isothermal atmosphere in which, the horizontal gradient of temperature would be zero –; a geostrophic wind that turns clockwise with respect to height (veering) is associated with more specific humidity advection. Conversely, as shown in figure 14; in the above-mentioned air, counterclockwise turning (backing) of the geostrophic wind with respect to height, implies less specific humidity advection by the geostrophic wind in the layer.

It is therefore possible to obtain a reasonable estimate of the mean specific humidity advection of the atmospheric layer between two near upper air stations from data on the vertical profile of the wind given by soundings at those upper air stations. Alternatively, the mean geostrophic wind at two points of one level of the atmospheric layer between two near upper air stations can be estimated from the mean field of specific humidity advection of that layer provided that the mean geostrophic wind velocity is known at other level. Thus, for example, if the mean geostrophic wind velocity at 850 hPa is known from two near upper air stations and the mean specific humidity advection in the layer 850 – 500 hPa between two near upper air stations is also known; the first prominent type of the moist wind equation can be applied to obtain the mean geostrophic wind velocity in the layer at 500 hPa.

First prominent type of moist wind vector, i.e., equation (4-M-I) has a simple form by integration with respect to vertical axis, as following:

$$\mathbb{v}_{\mathrm{M_I}} = -(0.608)\frac{R_d}{f}\langle T\rangle\left(\mathbb{k}_p \times \nabla_p\langle q\rangle\right)\ln\left(\frac{p_2}{p_1}\right) \quad (21\text{-M-I})$$

where in equation (21-M-I), $\langle\ldots\rangle$ is vertical averaging of phrase or parameter.

Analogous to simplification of equation (4-M-I); we can obtain simple form of equations (5-M-I-a) and (5-M-I-b) those show simple forms for components of first prominent type of moist wind vector for humid air with conditions (42); those are:

$$u_{\mathrm{M_I}} = (0.608)\frac{R_d}{f}\langle T\rangle\frac{\partial\langle q\rangle}{\partial y}\ln\left(\frac{p_2}{p_1}\right) \quad (22\text{-M-I-a})$$

and:

$$v_{\mathrm{M_I}} = -(0.608)\frac{R_d}{f}\langle T\rangle\frac{\partial\langle q\rangle}{\partial x}\ln\left(\frac{p_2}{p_1}\right) \quad (22\text{-M-I-b})$$

Again, in equations (22-M-I-a) and (22-M-I-b), $\langle\ldots\rangle$ is vertical averaging of phrase or parameter and furthermore, in equation (22-M-I-a) $u_{\mathrm{M_I}}$ is eastward component of moist wind by first prominent type and in equation (22-M-I-b) $v_{\mathrm{M_I}}$ is northward component of moist wind in first prominent type.

2.2.2. Moist wind; Approach by Geopotential

Conditions (42) is opposite to conditions (10), (11) and (12).

By considering conditions (12); if we apply the same process to derive equation (4-T-IV) here plus bearing in mind conditions (42); we get the other form of moist wind vector named second prominent type of moist wind, as we got in the case of dense wind i.e.:

$$\mathbb{v}_{\mathrm{M_{II}}} = \frac{1}{f}\mathbb{k}_p \times \nabla_p(\Phi_2 - \Phi_1) \quad (4\text{-M-II})$$

where in equation (4-M-II); $\mathbb{v}_{\mathrm{M_{II}}}$ stands for *second prominent type of moist wind vector*, f is Coriolis parameter, $\mathbb{k}_p$ is vertical unit vector in pressure coordinates system, ∇_p stands for gradient operator in pressure coordinates system, Φ_2 is geopotential at upper level of the atmospheric layer and Φ_1 is geopotential at lower level of the atmospheric layer.

Equation (4-M-II) is another version of the moist wind and states that: *second prominent type of moist wind is proportional to horizontal gradient of the thickness of the layer* and this subject is true if the variation of the thickness of the layer would produce only with the horizontal gradient of specific humidity.

From equation (4-M-II) we can derive eastward and northward components of second prominent type of moist wind easily:

$$u_{\mathrm{M_{II}}} = -\frac{1}{f}\frac{\partial}{\partial y}(\Phi_2 - \Phi_1) \quad (5\text{-M-II-a})$$

$$v_{\mathrm{M_{II}}} = \frac{1}{f}\frac{\partial}{\partial x}(\Phi_2 - \Phi_1) \quad (5\text{-M-II-b})$$

where in equation (5-M-II-a), $u_{\mathrm{M_{II}}}$ is eastward component of second prominent type of the moist wind and $v_{\mathrm{M_{II}}}$ is northward component of second prominent type of moist wind.

From second prominent type of moist wind vector or its components; one can find out that:

A: If we go from pole to equator, second prominent type of moist wind becomes stronger (with pay attention to footnote No. 7);

B: If the horizontal gradient of thickness of the layer would be greater, second prominent type of moist wind becomes stronger, because the moist wind is proportional to the horizontal gradient of thickness of the atmospheric layer and:

C: If the geopotential difference between lower level and upper level of the atmospheric layer will be higher, moist wind becomes more powerful.

In this manner; representative of second prominent type of moist wind vector, i.e., equation (4-M-II) with consideration of condition (42) shows that: "*second prominent type of moist wind blows parallel to isopleth of thickness, so that, more thickness of the atmospheric layer is located at the right side of downwind and less thickness of the atmospheric layer is located at the left side of downwind.*" (In the northern hemisphere) This fact is illustrated in figures 15 and 16.

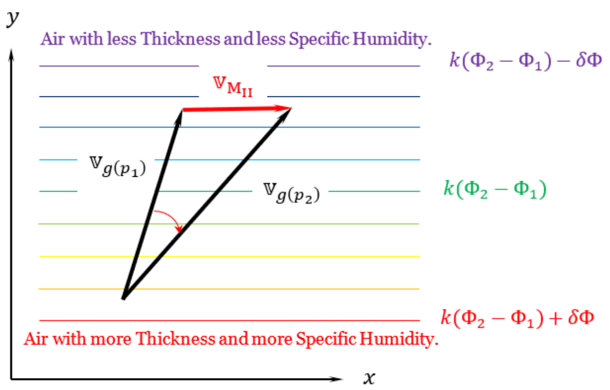

Figure 15. Relationship between clockwise turning of the geostrophic wind with respect to height (veering) and more thickness and more specific humidity advection. In the case; proportion coefficient k is: $k = \frac{1}{f}$

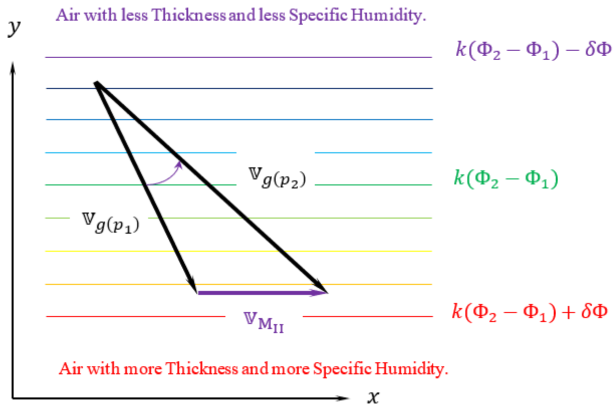

Figure 16. Relationship between counterclockwise turning of the geostrophic wind with respect to height (backing) and less thickness and less specific humidity advection. In the case; proportion coefficient k is: $k = \frac{1}{f}$

As it is shown in figure 15; a geostrophic wind that turns clockwise with respect to height (veering) is associated with more thickness and more specific humidity advection. Conversely, as shown in figure 16; counterclockwise turning (backing) of the geostrophic wind with respect to height, implies less thickness and less specific humidity advection by the geostrophic wind in the layer.
It is therefore possible to obtain a reasonable estimate of the mean thickness and specific humidity change of the atmospheric layer between two near upper air stations from data on the vertical profile of the wind given by soundings at those upper air stations. Alternatively, the mean geostrophic wind at two points of one level of the atmospheric layer between two near upper air stations can be estimated from the mean thickness and specific humidity field advection of that layer provided that the mean geostrophic wind velocity is known at other level. Thus, for example, if the mean geostrophic wind velocity at 850 hPa is known from two near upper air stations and the mean gradient of the thickness and changes of specific humidity in the layer 850 – 500 hPa between two near upper air stations are also known; the second prominent type of moist wind equation can be applied to obtain the mean geostrophic wind velocity in the layer at 500 hPa.
Therefore; the moist wind equation is an extremely useful diagnostic tool, which can use to check analyses of the observed wind and specific humidity fields for consistency; in the regions – same tropics – that environment is extremely isothermal[10].

3. Attention for similarities

There are some similarities in this work.
You realize that formulae (4-D-III), (4-T-IV) and (4-M-II) are alike.
Although, figure 9 – related to formula (4-T-IV) –, figure 15 – related to formula (4-M-II) – and figure related to formula (4-D-III) – that introduced in part I of this research – are alike.
Furthermore, figure 10 – related to formula (4-T-IV) – figure 16 – related to formula (4-M-II) – and figure related to formula (4-D-III) – that introduced in part I of this research – are alike.
Despite of above-mentioned similarities; there are basic discrepancies between them that will be explain in following paragraphs.
Formula (4-D-III) and related figures about it – those were presented in part I of this research –, were arisen from baroclinic weather that baroclinity of it, is related to both horizontal gradient of temperature and horizontal gradient of humidity.
Formula (4-T-IV), figure 9 and figure 10 were arisen from baroclinic weather that baroclinity of it, is related to horizontal gradient of temperature solely.
And formula (4-M-II) and figures 15 and 16 were arisen from baroclinic weather that baroclinity of it, were arisen from horizontal gradient of specific humidity solely.
Therefore; although – in above-mentioned formulae and figures –, there are similarities apparently; but in interior of them, there are striking differences between them.

4. Results and Discussion

All versions of dense wind equations, thermal wind equations and moist wind equations; are an extremely useful diagnostic tools, which is often used to check analyses of the observed wind field for consistency.
The proposition of dense wind, thermal wind and moist wind, those are propounded for the first time with these kind configurations; can help to better understanding of atmosphere dynamics. Furthermore, knowing the

[10] Note that here; we focus on isothermal environment; otherwise, the geostrophic wind has no physical meaning at equator or the regions near it. Although, the proposition of moist wind is valid there, and that is vectorial difference between wind in the vertical direction with some difference.

advection of the warm air, cold air, moist air, dry air or thickness of the atmospheric layer with exclusive specifications; can help to improve weather forecasting. Although, we referred to the variation of the geostrophic wind with respect to height here; but this proposition is valid for the variation of real wind in vertical direction with some modifications that needs separate discussion.

Thermal wind, moist wind or – as exist in the nature of the whole atmosphere – except somewhere that thermal or moist wind is true – dense wind; is struggle of the atmosphere to return thermodynamic equilibrium and complete the dynamic cycle of atmosphere.

As we noticed in part I of the research; this movement begins from the fact that solar radiation disturbs the thermodynamic equilibrium, resulting production of horizontal gradient of density. Horizontal gradient of density produces horizontal gradient of potential energy and in turn, this condition forces atmosphere to generate horizontal gradient of pressure gradian force and finally, it causes blow wind, for returning thermodynamic equilibrium of the atmosphere.

Due to various incoming solar radiation and non-uniform transfer of heat flux and humidity flux and diffusion of them from below of different layers of the atmosphere; atmosphere will be baroclinic and horizontal gradients of density are not same in the layers and wind velocities can't be the same at all atmospheric layers i.e., vertical shear of the wind. Therefore, this phenomenon produces dense wind that is effort of atmosphere to return into its thermodynamic equilibrium and reducing dense wind speed. By continuous reduction of dense wind speed, thermodynamic disequilibrium of atmosphere weakens and weakens, until reaching of thermodynamic equilibrium of the atmosphere. If we assume there will be no more solar radiation, finally the wind will be disappeared in presence of friction.

So, dense wind is the key of understanding for dense or light air, virtual temperature or thickness of the atmospheric layer advections and is one of the mechanisms of returning thermodynamic equilibrium of atmosphere.

And the same conditions are true for thermal wind and moist wind under their own situations.

5. Conclusions

This point is important that scale analysis can determine at which region the moist wind governs rather than thermal wind and vise-versa and where we can't apply any of them and we ought to use dense wind. Perhaps, this is the best result from the point that: *Looking at the variation of the geostrophic wind with respect to height – in general –, shouldn't limit to dry atmosphere and we expect only one form of thermal wind from this process, because atmosphere has humidity; as well as we shouldn't conclude moist wind from it, because – in general – atmosphere is not isothermal; because in general the air is not dry, as well as it is not isothermal in horizontal direction therefore, the variation of the geostrophic wind with respect to height should be describe with better tool, namely dense wind.*

Also, we have noticed in part I of this research; "It is necessary to note two basic points. First; until God wills and sun radiates; atmospheric and oceanic mediums are baroclinic. And the theory of barotropic medium – same as geostrophic wind – is acceptable for simplification of meteorological and oceanic analyses. Second point is with regard to more affection of temperature in variation of air density; although in many places far from oceanic medium, the dense wind gains energy more from horizontal gradient of temperature rather than the horizontal gradient of humidity, but it is not sufficient reason to call the vectorial difference of the geostrophic wind at upper and lower level of the atmospheric layer as thermal wind, because we can't deny presence of humidity anywhere."

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