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A New Look at the Vertical Shear of the Geostrophic Wind Part I: Dense Wind

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ARTICLE INFO

Article History:
Received: 23 Oct. 2022
Accepted: 16 Jan. 2023

Keywords:
Barotropic medium
Horizontal variation of density
Baroclinic medium
Thermal wind
Dense wind

ABSTRACT

From one point of view; we can divide atmosphere into two mediums. Barotropic medium, that in this medium, density doesn't change in horizontal direction and isobaric surfaces are parallel to each other in vertical direction. This medium can be motionless, but if in this medium, motion would be taken place, geostrophic wind doesn't change with respect to height.

On the other hand, the baroclinic medium has horizontal gradient of density, and causes various horizontal gradient of pressure with respect to height and implies various horizontal velocity at different levels of the atmosphere. Therefore; geostrophic wind varies with respect to height in this medium.

The horizontal gradient of density not only would produce by horizontal gradient of temperature, but also by horizontal gradient of humidity or combination of both. If horizontal gradient of density would be by both horizontal gradient of temperature and horizontal gradient of humidity – as in natural air, not in dry air – in the case; we name vectorial difference of geostrophic wind with respect to height; *dense wind*.

The purpose of this paper is introduction of three versions of dense wind in natural medium of air, not dry air. Basic axis of first version of dense wind is founded by density, second by virtual temperature and third one by thickness of atmospheric layer. Formulae related to each version is derived and every one of them, represents effects of one type of variation of geostrophic wind with respect to height. First version exhibits advection of light or dense air, second represents virtual temperature advection and third one demonstrates advection of thickness in atmospheric layer. Dense wind is powerful tool for consistency of wind field. Therefore, because air is not dry, the variation of the geostrophic wind with respect to height should be describe with better tool, namely *dense wind*.

1. Introduction

Variation of wind with respect to height has been investigated many times and, in this field, observational studies as well as researches have been done since the beginning of the last century. There are various profits for research on variation of the wind with respect to height. In the first place, it is used for detection of advection of light or dense air, and in the second place, it helps to detect of the thunder storm's type. The reason for second profit is related to the fact that second critical factor in determination of the type and the potential of intensity of thunder storm, is variation of wind with respect to height. However, for this matter of fact, some meteorologists focused their studies on planetary boundary layer, whereas others worked on troposphere as whole.

Regarding to first matter; observations of Charnock et al. [1] of trade wind, had been done at Anegada Island ($15^{\circ}N.64^{\circ}W$). This island, idiomatically, is isolated point of the earth that surrounded by ocean. Charnock and his colleagues' observations include 466 double soundings with balloon using theodolite in fifteen separate days during 27 days' observations at boundary layer. The important results in cases of Anegada's studies are as follows:

A: At the first 1350-meter (height) wind has 24 degrees veering;

B: Acceleration terms in comparison with Coriolis, pressure gradient force and friction terms; are ignorable;

C: They realized that observational wind at upwind coast is almost representative of wind over ocean considering perturbing effect of the Island.
Carlstead [2], tried to apply geostrophic wind's vertical profile in numerical model to cloudiness forecasting and determination of rainy area. Following Charnock et al. [1], variation of wind has been researched by Estoque [3] during study of planetary boundary layer at Christmas Island.
Moreover, one result of variation of geostrophic wind with respect to height, entitled *Thermal wind*, investigated widely, in baroclinic medium by Foster and Levy [4]. They studied variation of speed of geostrophic wind with respect to height for the reason of horizontal gradient of temperature and friction. Also, in their work, the ratio of geostrophic wind's speed has been investigated at two special levels.
Sometime, research on variation of wind with respect to height, is a tool for study of Temperature or humidity fluxes. And sometimes; it is an implement for search of momentum transfer. For the reason of the fact that atmospheric system or oceanic system are systems that, all their parameters are related to each other; therefore, investigation of variation of wind with respect to height, is usable for many purposes and applications undoubtedly.
All researchers those have been worked on variation of geostrophic wind with respect to height, had two common ideas. They have been called difference between two geostrophic wind vectors at two pressure levels; the *thermal wind* as the first idea. Also, they have been assumed the atmosphere as dry air, as a second idea.
Especially, this subject with same hypothesizes, has been used in dynamic meteorology's text books, that is to say, in introducing of thermal wind, they assumed atmosphere is dry and derived formulae related to the subject in this case; although this assumption used for simplicity of the work. For instance, the subject is written in Hess's text book [5], Gill's text book [6], Dutton's text book [7], Holton and Hakim's text book [8] and in other dynamic meteorology text books. In addition; thermal wind has an entrée in *Glossary of Meteorology* [9] with same descriptions.
The purpose of this paper is introducing of three versions of dense wind in natural medium of air that includes humidity, not dry air by mathematical approach.
In natural medium of air; variation of geostrophic wind with respect to height will be occurred, whenever, the field would be baroclinic medium and it is clear that the reason of baroclinity is "*the existence of horizontal gradient of density.*" For atmosphere; the horizontal gradient of density is related to the horizontal gradient of temperature as well as the horizontal gradient of humidity or in general; both of them. In troposphere, especially in planetary boundary layer – that can be extended until three thousand meter above the oceans' surface – there is the horizontal gradient of density because of existence of horizontal gradient of temperature, horizontal gradient of humidity or both of them; so, the field is baroclinic. Therefore; it is necessary that variation of geostrophic wind with respect to height would be study in real atmosphere carefully, i.e., atmosphere with humid air. Our aim in this paper is, looking to this subject in the condition of real atmosphere.
However, a question that arises from the above-mentioned introduction is; if we consider atmosphere as natural atmosphere including humidity; then how the looking to variation of the wind in vertical direction should be modify? In this paper, the variation of the geostrophic wind with respect to height will be considered in the natural atmosphere, not in dry air.

2. Geostrophic Wind

Geostrophic wind can be introduced by:

$$\mathbb{v}_g = f^{-1}\mathbb{k}_p \times \nabla_p \Phi \quad (1)$$

where in equation (1) $\mathbb{v}_g$ is geostrophic wind vector, f is Coriolis parameter, $\mathbb{k}$ is vertical unit vector in pressure coordinates system[1], ∇ is operator for gradient, Φ is geopotential and subscript p shows that equation (1) is written in pressure coordinates system.
Equation (1) shows the magnitude of geostrophic wind is proportional to the horizontal gradient of geopotential and is parallel to equipotential lines on isobaric surface.[8]
Writing eastward and northward components of geostrophic wind yields following equations:

$$u_g = -\frac{1}{f}\frac{\partial \Phi}{\partial y} \quad (2\text{-a})$$

and

$$v_g = \frac{1}{f}\frac{\partial \Phi}{\partial x} \quad (2\text{-b})$$

where in (2-a) u_g is eastward component of geostrophic wind or geostrophic current in ocean and in (2-b) v_g is northward component of geostrophic wind or geostrophic current in ocean.
By scale analysis of vertical component of momentum equation in midlatitudes and Cartesian coordinate system; we get hydrostatic approximation:

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} - g \cong 0 \quad (3)$$

In equation (3) ρ is density, p is pressure, g is acceleration due to gravity and z is vertical coordinate

[1] In this paper; whenever we refer to *"pressure coordinates system"* our purpose is "*Cartesian coordinates system with pressure as vertical coordinate"*. Note this coordinates system is "*Left-handed system*".

of Cartesian coordinates system. In practice, most of researchers use equation (3) as hydrostatic equation same below:

$$\frac{dp}{dz} = -\rho g \quad (4)$$

Multiplying both sides of equation (4) by dz we get:

$$dp = -\rho g dz \quad (5)$$

Consider a layer of air that its lower level has p_1 pressure and $z_1$height; and its upper level has p_2 pressure and z_2 height. By integrating equation (5) from lower level to upper level of the layer, we get:

$$(p_1 - p_2) = \bar{\rho} g (z_2 - z_1) \quad (6)$$

Where $\bar{\rho}$ is average density of the layer and by means of neglecting variation of acceleration due to gravity for meteorological purposes. Noting that $p_2 < p_1$ and $z_1 < z_2$; if we call $(p_1 - p_2) = \delta p$, the partial pressure of column of air in the layer; and $(z_2 - z_1) = \delta z$, the partial height or thickness of the layer; from equation (6) and these assumptions, we have:

$$\delta p = \bar{\rho} g \delta z \quad (7)$$

Calculation of pressure in equation (7) shows height is function of pressure, density and acceleration due to gravity, same below:

$$\delta z = {}^{\delta p}/_{\bar{\rho} g} \quad (8)$$

By means of equation (8) whenever we pass on isobaric surface; increasing $\bar{\rho}$ decreases δz and decreasing $\bar{\rho}$ increases δz. Therefore, for the reason of variation of temperature's horizontal gradient or humidity's horizontal gradient on isobaric surface; there is the horizontal gradient of height, because according to the equation of state for moist air – that will introduce afterwards – density is a function of pressure, temperature and humidity.
The definition of geopotential in Cartesian coordinates system is [8]:

$$\Phi = gz \quad (9)$$

where Φ is geopotential. Geopotential is multiplication of height and acceleration due to gravity; therefore – in the case – geopotential varies in vertical direction because of variation of height, and geostrophic wind must have vertical shear in the presence of a horizontal density gradient, as can be shown easily from simple physical considerations based on hydrostatic equilibrium; then, geostrophic wind varies in vertical direction. Since the geostrophic wind [equation (1)] is proportional to the geopotential gradient on an isobaric surface. Therefore, a geostrophic wind directed along the positive y axis that increases in magnitude with height, requires the slop of the isobaric surface along the x axis for the reason of increasing height as well, as shown in Figure 1. According to the equation (8), δz – the thickness of isobaric layer – corresponds to variation of density.

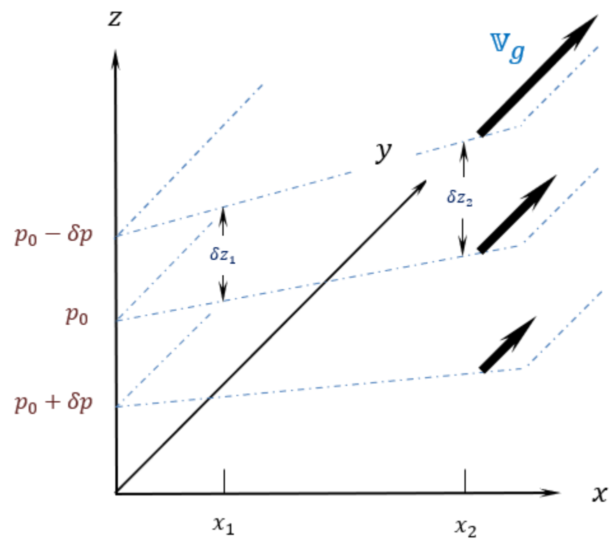

Figure 1. Relationship between vertical shear of the geostrophic wind and horizontal height gradients. (Note that $0 < \delta p$) [8]

3. Dense wind

The characteristic of baroclinic field, is that wind changes with respect to height, because of existence of the horizontal gradient of density. In this field, the geostrophic wind varies with height as well, as we can see in Figure 1.

3.1. Definition

"*Dense wind is vectorial difference of geostrophic wind vector at upper level and geostrophic wind vector at lower level*" (of the atmospheric layer), that is:

$$\mathbb{V}_{\mathrm{D}} \equiv \mathbb{V}_{g(p_2)} - \mathbb{V}_{g(p_1)} \quad (10)$$

where in Equation (10) $\mathbb{V}_{\mathrm{D}}$ stands for dense wind vector, $\mathbb{V}_g$ is geostrophic wind, and subscripts p_1 and p_2 refer to pressure levels in the manner that level p_2 has more height than level p_1 i.e., $p_2 < p_1$.
According to definition (10); eastward and northward components of dense wind can be shown as following:

$$u_{\mathrm{D}} = u_{g(p_2)} - u_{g(p_1)} \quad (11\text{-a})$$

and

$$v_{\mathrm{D}} = v_{g(p_2)} - v_{g(p_1)} \quad (11\text{-b})$$

Typical layer of atmospheric system is shown in Figure 2.

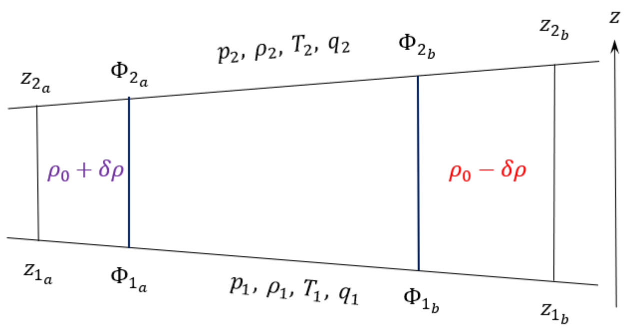

Figure 2. Typical layer of atmospheric system.

Despite its name, dense wind, while a vector, is not a true wind. Instead, it is a geostrophic wind shear, representing the change of wind with respect to height, causing some advections.

To derive dense wind equations, first consider the equation of state for humid air (common air) [10]. This equation has the form:

$$p = \rho_{\mathrm{M}}(1 + 0.608q)R_d T \tag{12}$$

where, p, ρ_{M}, q, R_d and T are air pressure, air density, specific humidity of air, gas constant for dry air and dry air temperature respectively.

If we introduce virtual temperature with following equation:

$$T_v = (1 + 0.608q)T \tag{13}$$

where, T_v is virtual temperature [11]; then the equation of the state for humid air can be written as:

$$p = \rho_{\mathrm{M}} R_d T_v \tag{14}$$

Where p is pressure, ρ_{M} is density of humid air, R_d is gas constant for dry air and T_v is virtual temperature.

Now, if we differentiate equations (2-a) and (2-b) with respect to p we get:

$$\frac{\partial u_g}{\partial p} = -\frac{1}{f}\frac{\partial}{\partial p}\left(\frac{\partial \Phi}{\partial y}\right) = -\frac{1}{f}\frac{\partial}{\partial y}\left(\frac{\partial \Phi}{\partial p}\right) \tag{15-a}$$

and

$$\frac{\partial v_g}{\partial p} = \frac{1}{f}\frac{\partial}{\partial p}\left(\frac{\partial \Phi}{\partial x}\right) = \frac{1}{f}\frac{\partial}{\partial x}\left(\frac{\partial \Phi}{\partial p}\right) \tag{15-b}$$

In pressure coordinates system, hydrostatic equation, applying for humid air and considering the equation of state; is [12]:

$$\frac{\partial \Phi}{\partial p} = -\alpha_{\mathrm{M}} = -\frac{1}{\rho_{\mathrm{M}}} = -\frac{R_d T_v}{p} \tag{16}$$

where α_{M} is specific volume of humid air and ρ_{M} is its density.

If we select equivalent of $\frac{\partial \Phi}{\partial p}$ from equation (16) that is $-\frac{1}{\rho_{\mathrm{M}}}$ and substitute in equations (15-a) and (15-b), we get:

$$\frac{\partial u_g}{\partial p} = -\frac{1}{f}\frac{\partial}{\partial y}\left(\frac{-1}{\rho_{\mathrm{M}}}\right) \tag{17-a}$$

and

$$\frac{\partial v_g}{\partial p} = \frac{1}{f}\frac{\partial}{\partial x}\left(\frac{-1}{\rho_{\mathrm{M}}}\right) \tag{17-b}$$

or

$$\frac{\partial u_g}{\partial p} = -\frac{1}{f(\rho_{\mathrm{M}})^2}\frac{\partial \rho_{\mathrm{M}}}{\partial y} \tag{18-a}$$

and

$$\frac{\partial v_g}{\partial p} = \frac{1}{f(\rho_{\mathrm{M}})^2}\frac{\partial \rho_{\mathrm{M}}}{\partial x} \tag{18-b}$$

which can be written as vector form:

$$\frac{\partial \mathbb{v}_g}{\partial p} = \frac{1}{f(\rho_{\mathrm{M}})^2}\mathbb{k}_p \times \nabla_p \rho_{\mathrm{M}} \tag{19-D-I}$$

we call the equation (19-D-I) as "*First version of Dense wind equation.*"

By integration of equation (19-D-I) from lower level p_1 to upper level p_2 ($p_2 < p_1$) of the atmospheric layer; one can derive "*First version of Dense wind vector*":

$$\mathbb{v}_{g(p_2)} - \mathbb{v}_{g(p_1)} \equiv \mathbb{v}_{\mathrm{D_I}} = \frac{1}{f}\int_{p_1}^{p_2}\left(\frac{1}{(\rho_{\mathrm{M}})^2}\mathbb{k}_p \times \nabla_p \rho_{\mathrm{M}}\right)dp \tag{20-D-I}$$

where in equation (20-D-I); $\mathbb{v}_{g(p_2)}$ is geostrophic wind vector at upper level of the atmospheric layer, $\mathbb{v}_{g(p_1)}$ is geostrophic wind vector at lower level of the atmospheric layer, $\mathbb{v}_{\mathrm{D_I}}$ stands for first version of dense wind vector, f is Coriolis parameter, p_1 is atmospheric pressure at lower level of the atmospheric layer, p_2 is atmospheric pressure at upper level of the atmospheric layer, ρ_{M} stands for density of natural air, $\mathbb{k}_p$ is vertical unit vector in pressure coordinates system and ∇_p is gradient operator in pressure coordinates system.

Eastward and northward components of first version of dense wind can be derived by integration of equations (18.a) and (18.b) – similar to deriving equation (20-D-I) – directly or one can determine the eastward and northward components of first version of dense wind from equation (20-D-I) directly:

$$u_{\mathrm{D_I}} = -\frac{1}{f}\int_{p_1}^{p_2}\left(\frac{1}{(\rho_{\mathrm{M}})^2}\frac{\partial \rho_{\mathrm{M}}}{\partial y}\right)dp \tag{21-D-I-a}$$

and

$$v_{\mathrm{D_I}} = \frac{1}{f}\int_{p_1}^{p_2}\left(\frac{1}{(\rho_{\mathrm{M}})^2}\frac{\partial \rho_{\mathrm{M}}}{\partial x}\right)dp \tag{21-D-I-b}$$

where in equation (21-D-I-a) $u_{\mathrm{D_I}}$ is eastward component of dense wind and in equation (21-D-I-b) $v_{\mathrm{D_I}}$ stands for northward component of dense wind, both of them from first version of dense wind.

From first version of dense wind vector or its components; one can find out that:

A: If we go from pole to equator, dense wind becomes stronger[2];

B: If the atmospheric layer has more altitude; then dense wind becomes more powerful;

C: If the horizontal gradient of density would be greater, dense wind becomes stronger, because dense wind is proportional to the horizontal gradient of density, and finally;

D: If the pressure difference will be higher in the layer, dense wind becomes more powerful.

In this manner, representative of first version of dense wind vector, i.e., equation (20-D-I) shows that: "*dense wind blows parallel to Isopycnals, so that, light air is located at the left side of downwind.*" (In the northern hemisphere) This fact is illustrated in Figures 3 and 4.

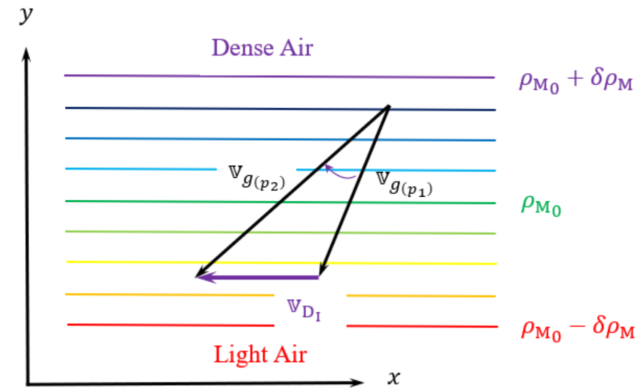

Figure 3. Clockwise rotating of geostrophic wind with respect to height; (Veering) and dense air advection.

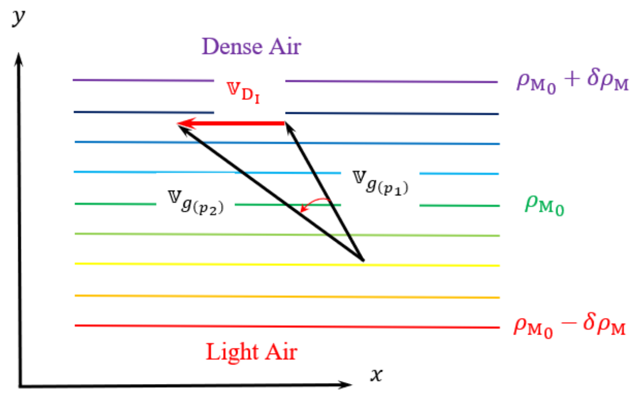

Figure 4. Counterclockwise rotation of geostrophic wind with respect to height (Backing) and light air advection.

In connection with Figure 3, clockwise turning of geostrophic wind with respect to height (veering) is associated with dense air advection by geostrophic wind in the layer. In other words; veering of geostrophic wind with respect to height associated with cold or dry air in the layer.

Conversely, as shown in Figure 4, counterclockwise turning of geostrophic wind with respect to height (backing) implies light air advection by geostrophic wind in the layer. In other words; backing of geostrophic wind with respect to height associated with advection of warm or moist air in the layer.

Therefore, it is possible to obtain a reasonable estimate of the horizontal light or dense air advection and its vertical dependence at a given location solely from data on the vertical profile of the wind given by a single sounding. Alternatively, the geostrophic wind at any level can be estimated from the advection of light or dense air field, provided that the geostrophic velocity is known at a single level.

Thus, for example, if the geostrophic wind at 850 h Pa is known and the mean horizontal density gradient in the layer 850 – 500 h Pa is also known, the first version of thermal wind equation can be applied to obtain the geostrophic wind at 500 h Pa.

Likewise, it is possible to introduce simpler forms of first version of dense wind vector and its components. Among of them there are:

$$\mathbb{v}_{D_I} = \frac{1}{f}\left\langle\frac{1}{(\rho_M)^2}\right\rangle \int_{p_1}^{p_2} \left(\mathbb{k}_p \times \nabla_p \rho_M\right) dp \qquad \text{(22-D-I)}$$

with eastward component:

$$u_{D_I} = -\frac{1}{f}\left\langle\frac{1}{(\rho_M)^2}\right\rangle \int_{p_1}^{p_2} \frac{\partial \rho_M}{\partial y}\, dp \qquad \text{(23-D-I-a)}$$

and northward component:

$$v_{D_I} = \frac{1}{f}\left\langle\frac{1}{(\rho_M)^2}\right\rangle \int_{p_1}^{p_2} \frac{\partial \rho_M}{\partial x}\, dp \qquad \text{(23-D-I-b)}$$

Here the angle brackets in equations (22-D-I), (23-D-I-a) and (23-D-I-b) denote a vertical average.[3]

Even, one can introduces simpler form of first version of dense wind vector and its components same below:

$$\mathbb{v}_{D_I} = \frac{1}{f}\left\langle\frac{1}{(\rho_M)^2}\right\rangle \langle \mathbb{k}_p \times \nabla_p \rho_M \rangle (p_2 - p_1) \qquad \text{(24-D-I)}$$

with eastward component:

$$u_{D_I} = -\frac{1}{f}\left\langle\frac{1}{(\rho_M)^2}\right\rangle \left(\frac{\partial \langle\rho_M\rangle}{\partial y}\right)_p (p_2 - p_1) \qquad \text{(25-D-I-a)}$$

and northward component:

$$v_{D_I} = \frac{1}{f}\left\langle\frac{1}{(\rho_M)^2}\right\rangle \left(\frac{\partial \langle\rho_M\rangle}{\partial x}\right)_p (p_2 - p_1). \qquad \text{(25-D-I-b)}$$

[2] Use of geostrophic wind in tropical regions must be with careful deliberation because geostrophic wind in these regions is magnified and especially on equator is meaningless.

[3] There are many procedures about averaging in meteorology. Every method for averaging particular parameter or term, is related to position, time and specifications of the medium. All meteorological or oceanographical parameters; decreases logarithmic or semi-logarithmic and increases exponentially or semi-exponentially with respect to height or depth. Therefore, for every case, we need to use special manner for it. As a simple example, if we consider a layer of air between 500 and 300 hectopascals, the easiest way is using linear averaging, i.e., adding the value of parameters or terms of above and below the layer and dividing the result by 2.

3.2. Second version of dense wind

It can be shown that if we choose equivalent of $\frac{\partial \Phi}{\partial p}$, $-\frac{R_d T_v}{p}$, instead of $-\frac{1}{\rho_{\mathrm{M}}}$ from equation (16) and substitute in equations (15-a) and (15-b); then differentiate these equations with respect to p, one can obtain another version of dense wind equation, dense wind vector and its components as following forms respectively:

$$\frac{\partial u_g}{\partial p} = -\frac{1}{f}\frac{\partial}{\partial y}\left(-\frac{R_d T_v}{p}\right) \qquad (26\text{-a})$$

and

$$\frac{\partial v_g}{\partial p} = \frac{1}{f}\frac{\partial}{\partial x}\left(-\frac{R_d T_v}{p}\right) \qquad (26\text{-b})$$

or

$$\frac{\partial u_g}{\partial \ln p} = \frac{R_d}{f}\frac{\partial T_{\mathrm{v}}}{\partial y} \qquad (27\text{-a})$$

and

$$\frac{\partial v_g}{\partial \ln p} = -\frac{R_d}{f}\frac{\partial T_v}{\partial x} \qquad (27\text{-b})$$

Equations (27-a) and (27-b) can be written in the vector form in the following line:

$$\frac{\partial \mathbb{v}_g}{\partial \ln p} = -\frac{R_d}{f}\left(\mathbb{k}_p \times \nabla_p T_v\right) \qquad (19\text{-D-II})$$

we call the equation (19-D-II) as "*second version of Dense wind equation.*"

By integration of equation (19-D-II) from lower level p_1 to upper level p_2 $(p_2 < p_1)$ of the atmospheric layer; one can derive "*second version of Dense wind vector*":

$$\mathbb{v}_{g(p_2)} - \mathbb{v}_{g(p_1)} \equiv \mathbb{v}_{\mathrm{D_{II}}} = -\frac{R_d}{f}\int_{p_1}^{p_2}\left(\mathbb{k}_p \times \nabla_p T_v\right) d \ln p \qquad (20\text{-D-II})$$

where in equation (20-D-II); $\mathbb{v}_{g(p_2)}$ is geostrophic wind vector at the upper level of the atmospheric layer, $\mathbb{v}_{g(p_1)}$ is geostrophic wind vector at the lower level of the atmospheric layer, $\mathbb{v}_{\mathrm{D_{II}}}$ stands for second version of dense wind vector, f is Coriolis parameter, p_1 is atmospheric pressure at the lower level of the atmospheric layer, p_2 is atmospheric pressure at the upper level of the atmospheric layer, $\mathbb{k}_p$ is vertical unit vector in pressure coordinates system, ∇_p is gradient operator in pressure coordinates system and T_{v} stands for virtual temperature of natural air.

Eastward and northward components of second version of dense wind in this point of view, can be derived by integration of equations (27.a) and (27.b) directly or decompose of second version of dense wind vector (20-D-II) presently:

$$u_{g(p_2)} - u_{g(p_1)} \equiv u_{\mathrm{D_{II}}} = \frac{R_d}{f}\int_{p_1}^{p_2}\frac{\partial T_{\mathrm{v}}}{\partial y} d \ln p \qquad (21\text{-D-II-a})$$

and

$$v_{g(p_2)} - v_{g(p_1)} \equiv v_{\mathrm{D_{II}}} = -\frac{R_d}{f}\int_{p_1}^{p_2}\frac{\partial T_{\mathrm{v}}}{\partial x} d \ln p \qquad (21\text{-D-II-b})$$

where in equation (21-D-II-a) $u_{\mathrm{D_{II}}}$ is eastward component of second version of dense wind and in equation (21-D-II-b) $v_{\mathrm{D_{II}}}$ stands for northward component of second version of dense wind.

From second version of dense wind vector or its components; one can find out that:

A: If we go from pole to equator, dense wind becomes stronger (with pay attention to footnote No. 2);

B: If the horizontal gradient of virtual temperature would be greater, dense wind becomes stronger, because, dense wind is proportional to the horizontal gradient of virtual temperature, and finally;

C: If the pressure difference will be higher in the layer, dense wind becomes more powerful.

In this manner, representative of second version of dense wind vector, i.e., equation (20-D-II) shows that: "*dense wind blows parallel to Isopleths of virtual temperature so that, greater virtual temperature is located at the right side of downwind.*" (In the northern hemisphere) This is illustrated in Figures 5 and 6.

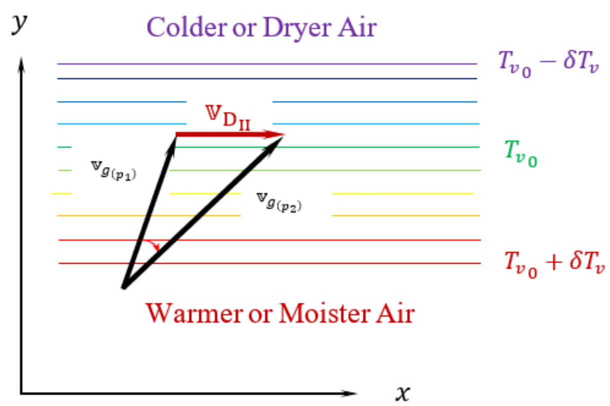

Figure 5: Clockwise rotation of geostrophic wind with respect to height. (Veering) and warmer or moister air advection.

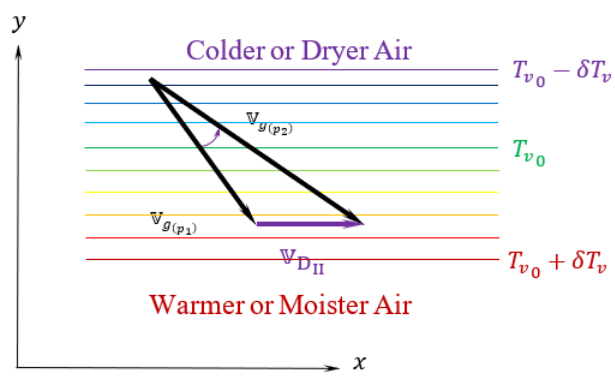

Figure 6: Counterclockwise rotation of geostrophic wind with respect to height. (Backing) and colder or dryer air advection.

In connection with Figure 5; clockwise turning of geostrophic wind with respect to height (veering) is associated with warmer or moister air advection by geostrophic wind in the layer. Conversely, as shown in Figure 6; counterclockwise turning of geostrophic wind with respect to height (backing) implies colder or dryer air advection by geostrophic wind in the layer.

In a similar way, it is possible to introduce simpler forms of second version of dense wind vector and its components in the following lines:

$$\mathbb{v}_{\mathrm{D_{II}}} = -\frac{R_d}{f}\langle \mathbb{k}_p \times \nabla_p T_v\rangle \ln\left(\frac{p_2}{p_1}\right) \tag{22-D-II}$$

$$u_{\mathrm{D_{II}}} = \frac{R_d}{f}\frac{\partial\langle T_v\rangle}{\partial y}\ln\left(\frac{p_2}{p_1}\right) \tag{25-D-II-a}$$

$$v_{\mathrm{D_{II}}} = -\frac{R_d}{f}\frac{\partial\langle T_v\rangle}{\partial x}\ln\left(\frac{p_2}{p_1}\right) \tag{25-D-II-b}$$

In equations (22-D-II), (25-D-II-a) and (25-D-II-b); broken brackets refer to average value of term or parameter in the atmospheric layer.

Therefore, it is possible to obtain a reasonable estimate of the horizontal mean virtual temperature advection and its vertical dependence at a given location solely from data on the vertical profile of the wind given by a single sounding. Alternatively, the geostrophic wind at any level can be estimated from the advection of mean virtual temperature field, provided that the geostrophic velocity is known at a single level. Thus, for example, if the geostrophic wind at 850 hectopascals level is known and the mean horizontal virtual temperature gradient in the layer of 850–500 hectopascals is also known, the second dense wind equation can be applied to obtain the geostrophic wind at 500 hectopascals.

3.3. Third version of dense wind

Furthermore; there is another version of dense wind. This version of dense wind is originated form the thickness of the atmospheric layer.

Consider hydrostatic equation in Cartesian coordinates system:

$$\frac{dp}{dz} = -\rho g \tag{4}$$

Parameters in equation (4) have been introduced after equation (3). By considering equation (9); equation (4) can be written:

$$-\frac{dp}{\rho} = g dz = d\Phi \tag{28}$$

where $d\Phi$ is differential part of geopotential. Atmosphere has humidity, therefore; equation (28) for moist air can be written as:

$$d\Phi = -\frac{dp}{\rho_{\mathrm{M}}} \tag{29}$$

where ρ_{M} stands for density of (natural) air.

Consider equation of state for moist air [equation (14)], computation of density of moist air from equation (14) yields:

$$\rho_{\mathrm{M}} = \frac{p}{R_d T_v} \tag{30}$$

If we substitute equivalent of ρ_{M} from equation (30) into equation (29), yields:

$$d\Phi = -R_d T_v \frac{dp}{p} = -R_d T_v d\ln p \tag{31}$$

Getting approximate increasing parts of equation (31), yields:

$$\delta\Phi = -R_d T_v\, \delta \ln p \tag{32}$$

If we integrate equation (31) with respect to vertical coordinate; from the below of atmospheric layer with pressure p_1 and geopotential $\Phi_1$to the upper layer of it with pressure p_2 and geopotential Φ_2 as Figure 2 shows; we get:

$$\int_{p_1}^{p_2} d\Phi = -R_d \int_{p_1}^{p_2} T_v d\ln p \tag{33}$$

After calculating equation (33), we have:

$$\delta\Phi \equiv (\Phi_2 - \Phi_1) = R_d \int_{p_2}^{p_1} T_v\, d\ln p \tag{34}$$

where $\delta\Phi$ is the thickness of the layer by geopotential and equation (34) shows that the thickness of the atmospheric layer is proportional to vertical average of virtual temperature when we consider an atmospheric layer. Therefore, we call it "*The thickness equation*".

Consider equations (15-a) and (15-b):

$$\frac{\partial u_g}{\partial p} = -\frac{1}{f}\frac{\partial}{\partial y}\left(\frac{\partial\Phi}{\partial p}\right) \tag{15-a}$$

and

$$\frac{\partial v_g}{\partial p} = \frac{1}{f}\frac{\partial}{\partial x}\left(\frac{\partial\Phi}{\partial p}\right) \tag{15-b}$$

And, equations (15-a) and (15-b) can be written in the vector form as bellow:

$$\frac{\partial \mathbb{v}_g}{\partial p} = \frac{1}{f}\mathbb{k}_p \times \nabla_p\left(\frac{\partial\Phi}{\partial p}\right) \tag{19-D-III}$$

Equation (19-D-III) is called "*third version of dense wind equation*."

By vertical integrating of equation (19-D-III) from the below level of the atmospheric layer with pressure p_1 and geopotential Φ_1 to the upper level of this atmospheric layer with pressure p_2 and geopotential Φ_2; we get:

$$\mathbb{v}_{g(p_2)} - \mathbb{v}_{g(p_1)} \equiv \mathbb{v}_{\mathrm{D_{III}}} = \frac{1}{f}\int_{p_1}^{p_2}\left[\mathbb{k}_p \times \nabla_p\left(\frac{\partial\Phi}{\partial p}\right)\right] dp \tag{35}$$

Therefore, the "*third version of dense wind vector*" is:

$$\mathbb{v}_{\mathrm{D_{III}}} = \frac{1}{f}\mathbb{k}_p \times \nabla_p(\Phi_2 - \Phi_1) \qquad \text{(20-D-III)}$$

The third version of dense wind shows dense wind is proportional to thickness gradient of the atmospheric layer.

Now, we can derive eastward and northward components of third version of dense wind from equation (20-D-III) directly or integrating equations (15-a) and (15-b) with respect to vertical coordinate; from the below of atmospheric layer with pressure p_1 and geopotential Φ_1 to the upper of atmospheric layer with pressure p_2 and geopotential Φ_2 as is shown in Figure 2; we get:

$$\int_{p_1}^{p_2}\frac{\partial u_g}{\partial p}\,dp = -\frac{1}{f}\int_{p_1}^{p_2}\frac{\partial}{\partial y}\left(\frac{\partial \Phi}{\partial p}\right)dp = -\frac{1}{f}\frac{\partial}{\partial y}\int_{p_1}^{p_2} d\Phi \qquad \text{(36-a)}$$

and

$$\int_{p_1}^{p_2}\frac{\partial v_g}{\partial p}\,dp = \frac{1}{f}\int_{p_1}^{p_2}\frac{\partial}{\partial x}\left(\frac{\partial \Phi}{\partial p}\right)dp = \frac{1}{f}\frac{\partial}{\partial x}\int_{p_1}^{p_2} d\Phi \qquad \text{(36-b)}$$

Calculation's results of above equations are as follows:

$$u_{g(p_2)} - u_{g(p_1)} \equiv u_{\mathrm{D_{III}}} = -\frac{1}{f}\frac{\partial}{\partial y}(\Phi_2 - \Phi_1) \qquad \text{(21-D-III-a)}$$

and

$$v_{g(p_2)} - v_{g(p_1)} \equiv v_{\mathrm{D_{III}}} = \frac{1}{f}\frac{\partial}{\partial x}(\Phi_2 - \Phi_1) \qquad \text{(21-D-III-b)}$$

From third version of dense wind vector or its components; one can find out that:

A: If we go from pole to equator, dense wind becomes stronger (with pay attention to footnote No. 2);

B: If the horizontal gradient of the thickness of atmospheric layer would be greater, dense wind becomes stronger, because, dense wind is proportional to the horizontal gradient of the thickness of the atmospheric layer, and finally;

C: If the geopotential difference between lower level and upper level of the atmospheric layer will be higher, dense wind becomes more powerful.

In this manner, representative of third version of dense wind vector, i.e., equation (20-D-III) shows that: "*dense wind blows parallel to isolines of thickness of the atmospheric layer so that, more thickness of the layer is located at the right side of downwind.*" (In the northern hemisphere) This is illustrated in Figures 7 and 8.

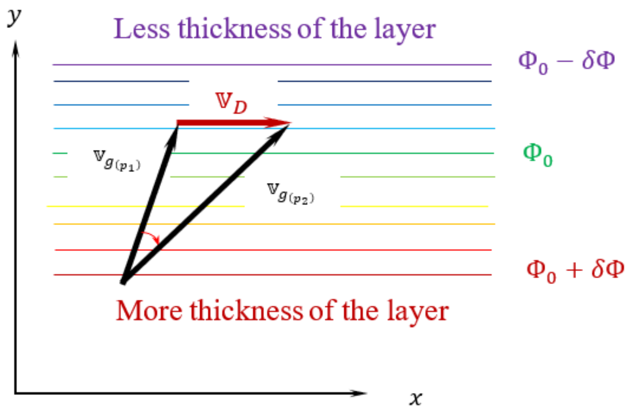

Figure 7: Clockwise rotation of geostrophic wind with respect to height. (Veering) and more thickness advection.

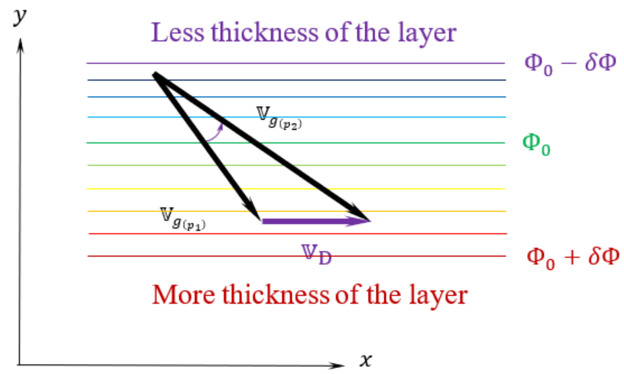

Figure 8: Counterclockwise rotation of geostrophic wind with respect to height. (Backing) and less thickness advection.

Therefore, it is possible to obtain a reasonable estimate of the horizontal more (or less) thickness advection of the atmospheric layer and its vertical dependence at a given location solely from data on the vertical profile of the wind given by a single sounding. Alternatively, the geostrophic wind at any level can be estimated from the advection of more (or less) thickness of the atmospheric layer, provided that the geostrophic velocity is known at a single level. Thus, for example, if the geostrophic wind at 700 hectopascals level is known and the more (or less) thickness advection in the layer 700–500 hectopascals is also known; the advection of more (or less) thickness, can be applied to third version of dense wind for obtain the geostrophic wind at 500 hectopascals level.

4. Results and Discussion

All versions of the dense wind equation are an extremely useful diagnostic tools, which is often used to check analyses of the observed wind field for consistency.

It can also be used to estimate the mean horizontal dense air, virtual temperature and thickness advections in an atmospheric layer as shown in Figures 3, 4, 5, 6, 7 and 8 respectively.

Dense wind – as exist in the nature of atmosphere except somewhere that thermal wind or moist wind is true – is struggle of the atmosphere to return thermodynamic equilibrium and complete the dynamic cycle of atmosphere. This movement begins from the fact that solar radiation disturbs the thermodynamic equilibrium resulting production of horizontal gradient of density. Horizontal gradient of density produces horizontal gradient of potential energy and in turn, this condition forces atmosphere to generate horizontal gradient of pressure and finally, it causes to blow wind for returning thermodynamic equilibrium of atmosphere.

Concerning various insolation and non-uniform transfer of diffusion of heat and humidity in the different layers of the atmosphere; horizontal gradients of density are not same in the layers and wind velocities cannot be the same at atmospheric layers. Therefore, this phenomenon produces dense wind that is effort of atmosphere to adjust horizontal gradient of density and reducing dense wind speed. By continuous reduction of dense wind speed, thermodynamic disequilibrium of atmosphere weakens and weakens, until returning of thermodynamic equilibrium of the atmosphere. If we assume there will be no more solar radiation, finally the wind will be disappeared in presence of friction.

So, dense wind is the key of understanding of dense or light air; virtual temperature or thickness of the atmospheric layer advections and one of the mechanisms of returning thermodynamic equilibrium of atmosphere.

5. Conclusions

Looking at the variation of the geostrophic wind with respect to height, shouldn't limit to dry atmosphere because the atmosphere has humidity and not dry and strictly in general, the air is not dry. Therefore, the variation of the geostrophic wind with respect to height should be describe with better tool, namely *dense wind*. It is necessary to note two basic points. First; until God wills and sun radiates; atmospheric and oceanic mediums are baroclinic. And the theory of barotropic medium – same as geostrophic wind – is acceptable for simplification of meteorological and oceanic analyses. Second point is with regard to more affection of temperature in variation of air density; although in many places far from oceanic medium, the dense wind gains energy more from horizontal gradient of temperature rather than the horizontal gradient of humidity, but it is not sufficient reason to call the vectorial difference of the geostrophic wind at upper and lower level of the layer as thermal wind, because we cannot deny presence of humidity anywhere.

List of Symbols

Symbol	Description
f	Coriolis parameter
g	Acceleration due to gravity
$\mathbb{k}_p$	Vertical unit vector in pressure coordinates system
p	Pressure, vertical coordinate of pressure coordinates system
p_0	Reference pressure
p_1	Pressure of below level of layer
p_2	Pressure of upper level of layer
q	Specific humidity
q_1	Specific humidity of lower level of layer
q_2	Specific humidity of upper level of layer
R_d	Gas constant for dry air
T	Temperature
T_1	Temperature of below level of layer
T_2	Temperature of upper level of layer
T_v	Virtual temperature
T_{v0}	Reference virtual temperature
u	Eastward component of velocity
u_{D}	Eastward component of dense wind
$u_{\mathrm{D_I}}$	1st Ver. of, eastward component of dense wind
$u_{\mathrm{D_{II}}}$	2nd Ver. of, eastward component of dense wind
$u_{\mathrm{D_{III}}}$	3rd Ver. of, eastward component of dense wind
u_g	Eastward component of geostrophic wind
v	Northward component of velocity
v_{D}	Northward component of dense wind
$v_{\mathrm{D_I}}$	1st Ver. of Northward component of dense wind
$v_{\mathrm{D_{II}}}$	2nd Ver. of Northward component of dense wind
$v_{\mathrm{D_{III}}}$	3rd Ver. of Northward component of dense wind
v_g	Northward component of geostrophic wind
$\mathbb{V}_{\mathrm{D}}$	Dense wind vector
$\mathbb{V}_{\mathrm{D_I}}$	1st Ver. of Dense wind vector
$\mathbb{V}_{\mathrm{D_{II}}}$	2nd Ver. of Dense wind vector
$\mathbb{V}_{\mathrm{D_{III}}}$	3rd Ver. of Dense wind vector
$\mathbb{V}_g$	Geostrophic wind vector
x	Eastward direction
x_1	First distance
x_2	Second distance
y	Northward direction
z	Height, vertical coordinate of Cartesian coordinates system
z_1	Height of below level of layer
z_2	Height of upper level of layer
α_{M}	Specific volume of humid air
δp	Increment of pressure
$\delta \ln p$	Increment of natural logarithm of pressure
δT_v	Increment of virtual temperature
δz	Increment of height
δz_1	Increment of height for first distance
δz_2	Increment of height for second distance
$\delta \rho$	Increment of density
$\delta \Phi$	Increment of geopotential

ρ	Density
ρ_0	Reference density
ρ_1	Density of lower level of layer
ρ_2	Density of upper level of layer
ρ_{M}	Density of humid air
$\bar{\rho}$	Average density
Φ	Geopotential
Φ_0	Reference geopotential
Φ_1	Geopotential of lower level of layer
Φ_2	Geopotential of upper level of layer
$\ln \ldots$	Natural logarithm
∇_p	Gradient operator in pressure coordinates system
$\langle \ldots \rangle$	Vertical average

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