INTERNATIONAL JOURNAL OF
COASTAL, OFFSHORE & ENVIRONMENTAL ENGINEERING
IJCOE Vol.7/No. 2/Spring 2022 (49-54)

Available online at: www. Ijcoe.org

Numerical modeling of underwater acoustic wave using Differential Quadrature Method

Abolfazal Delbari[1], Ali Sheykhbahaei[2*]

[1] *Department of Physical Oceanography, Faculty of Marine Science, Khorramshahr University of Marine Science and Technology, Khorramshahr, Iran, abolfazl_delbari@yahoo.com*
[2*] *Department of Nonliving Resources of Atmosphere and Ocean, Faculty of Marine Science and Technology, University of Hormozgan, Bandar Abbas, Iran Abbas, Iran, ali.sheykhbahai@gmail.com*

ARTICLE INFO

Received: 10 Sep. 2021
Accepted: 1 Mar. 2022

Keywords:
Acoustic Wave Propagation
Differential Quadrature Method
Numerical Modeling

ABSTRACT

In this paper underwater acoustic wave propagation is studied numerically by Differential Quadrature method. Numerical methods are different with respect to accuracy. computer costs and practical flexibility. In this study Differential Quadrature (DQ) method is applied for numerical solution of underwater acoustic wave for first time. Two experimental cases are used to validate the two-dimensional wave model. first the numerical results are verified by analytical solution and the second one showed the applicability of current method in complex domain. Comparisons demonstrate the efficiency. accuracy and robustness of the Differential Quadrature method for acoustic wave simulation.

1. Introduction

During the recent two decades studies on the mathematical modeling of underwater acoustic wave propagation have been established. Numerous techniques have been used in the past to model acoustic wave propagation. As indicated by [1] most of these methods can be classified as Ray methods. Normal Mode methods and Parabolic Equation methods. The present study focuses on the mathematical modeling of underwater acoustic propagation over irregular bottom topography with Parabolic Equation methods.

Different numerical methods that have been applied to solve acoustic wave equation[2,3]. Costa et al. developed a numerical model to simulate the 2D acoustic wave propagation in the vicinity of an underwater configuration which combines two sub-regions using the Boundary Element Method and the Method of Fundamental Solutions[4]. Murphy and Chin-Bing applied the finite element method to produce a full-wave range-dependent scalar ocean acoustic propagation model[5]. Keiswetter et al. developed a program which uses finite-difference techniques to approximate the solution of the two-dimensional heterogeneous acoustic wave equation[6].

In this study a Differential Quadrature (DQ) Method is applied for the first time to solve the acoustic wave equation. Bellman et al. introduced DQ numerical method as a simple and highly accurate numerical technique[7]. which approximates the derivatives of function using a weighted sum of all the functional values along the orthogonal axis. Shu developed the weighting coefficient as a high-order polynomial that can be computed by the Lagrange polynomial interpolation with a recurrence relationship[8]. From the literature review. numerical solution by the DQ method can be very precise using a simple mesh system with a high-order polynomial employed. However, due to the geometric relationship of the wanted and the referenced nodes. the DQ method cannot be applied directly to irregular-domain problems. Shu and his colleagues proposed a domain-free discretization method to try to improve this disadvantage [9,10]. Nevertheless. the domain-free discretization method involves some points which may not be mesh nodes in the physical domain. and the physical values at these points require an interpolation/extrapolation scheme to be obtained. Such a technique will introduce an additional error near the boundaries.

Meanwhile. solution of the acoustic wave equation in a large domain obtained a large system of linear equations which should be solved simultaneously because of the nature of an elliptic boundary value problem. Therefore direct methods like Gauss elimination for solving a system of linear equation require huge storage and CPU time. For this purpose the iterative scheme would be the best choice. In this study Conjugate Gradient (CG) iterative method is applied to guarantee convergence of solving the linear system of equations that was used by several researchers in the literature [11].

2. Mathematical Problem Statement

2.1. Governing Equation

In the vertical plane (x.z) for homogeneous harmonic two-dimensional problems. with assuming ideal fluid theory and constant sound velocity c. the complex velocity potential is defined as

$$\varphi(\mathrm{x.z.t}) = \phi(\mathrm{x.z}) exp(-j\omega t) \quad (1)$$

where ω is the acoustic wave frequency. ϕ is the complex potential amplitude($j = \sqrt{-1}$).
The wave equation then reduces to a Helmholtz equation in domain Ω

$$\nabla^2\phi + k^2\phi = -\sum_{i=1}^{N_S} S_i\delta\left(x_{S_i}, z_{S_i}\right) \text{ in } \Omega \quad (2)$$

$$\nabla^2\phi + k_0^2 n^2\phi = -\sum_{i-1}^{N_S} S_i\,\delta(x_{S_i}, z_{S_i}) \quad (3)$$

Where $k_0 = w/c_0$ is the reference wave number.

$$n = [c_0 / c](1 + i\frac{BETA}{54.575054}) \quad (4)$$

is index of refraction.(BETA=attenuation (dB/wavelength) and S_i are values denote (potential) strengths of N_s point sources at locations $\left(x_{S_i}, z_{S_i}\right)$ (Fig. 1).

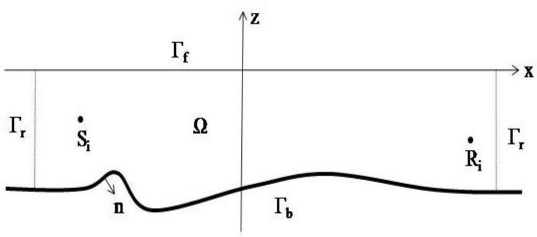


Figure 1. Schematic of underwater acoustic wave propagation domain

The relation between acoustic pressure and complex potential amplitude is

$$p(x,z) = \rho\omega^2\phi(x,z) \quad (5)$$

2.2. Boundary Conditions

For solving acoustic wave propagation appropriate boundary condition must be specified along boundary domain. The bottom is assumed rigid. hence; a no-flow condition applies

$$\frac{\partial\phi}{\partial n} = 0 \text{ on } \Gamma_b \text{ and } \Gamma_{b_i} \ (i = 1.2) \quad (6)$$

At the steady free surface. assuming atmospheric pressure p = 0. we obtain from the linearized Bernoulli equation

$$p = -\rho\frac{\partial\phi}{\partial t} = j\omega\rho_0\phi = 0 \quad (7)$$

with ρ_0 the fluid density. i.e. with no wave present. $\phi = 0$ on Γ_f and Γ_{f_i} (i = 1.2)
On open boundaries. Γ_{r_i} (i = 1. 2). a radiation condition is specified such that the potential and its normal gradient are continuous from inside the computational domain to outside

$$\phi^i = \phi \text{ on } \Gamma_{r_i} \ (i = 1.2) \quad (8)$$

$$\frac{\partial\phi^i}{\partial n} = \frac{\partial\phi}{\partial n} \text{ on } \Gamma_{r_i} \ (i = 1.2) \quad (9)$$

3. Numerical Descretization

3.1. Differential Quadrature Descretization

Differential quadrature is characterized by approximating the derivatives of a function using a weighted linear sum of all the functional values along the orthogonal axis.

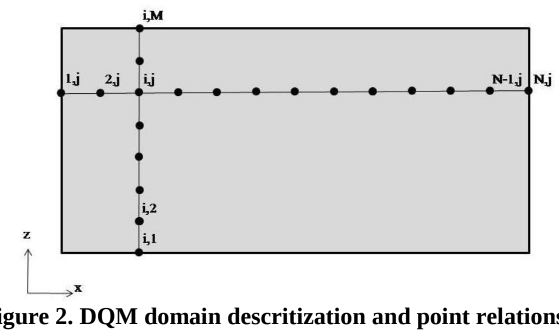


Figure 2. DQM domain descritization and point relationship

Therefore as shown in Fig.2. if the black point is a referenced point then it's derivative with respect to x (or y) direction is approximated by weighted linear sum of gray (or white) point function values that spread in all the problem domain. Then the DQ descretization of two-dimensional problem domain ($N \times M$) at any location (x_i, y_j) the n_{th} order derivative of a function $f(x,y)$ with respect to x and y can be approximated by

$$\frac{\partial^n f(x_i, y_j)}{\partial x^n} = \sum_{k=1}^{N} w_{i,k}^{(n)} f(x_k, y_j) \tag{10}$$

$$\frac{\partial^n f(x_i, y_j)}{\partial y^n} = \sum_{k=1}^{M} w_{j,k}^{(n)} f(x_i, y_k) \tag{11}$$

respectively. for $i = 1,2,\ldots,N$ and $j = 1,2,\ldots,M$ Where $w^{(n)}$ is the weighing coefficient of n_{th} order derivative. Based on Shu's general approach. the explicit formulas for the first order weighting coefficients for x direction are

$$w_{i,j}^{(1)} = \frac{\prod_{k=1,\, k\neq i}^{N} (x_i - x_k)}{(x_i - x_j) \cdot \prod_{k=1,\, k\neq j}^{N} (x_j - x_k)}, \qquad for \quad i \neq j \tag{12a}$$

$$w_{i,i}^{(1)} = -\sum_{j=1,\; j\neq i}^{N} w_{i,j}^{(1)} \tag{12b}$$

Also for the second order derivative weighting coefficient of Shu's general approach is:

$$w_{i,j}^{(2)} = 2w_{i,j}^{(1)} \cdot \left(w_{i,i}^{(1)} - \frac{1}{x_i - x_j} \right), \qquad for \quad i \neq j \tag{13a}$$

$$w_{i,i}^{(2)} = -\sum_{j=1,\; j\neq i}^{N} w_{i,j}^{(2)} \tag{13b}$$

Similarly for y direction. weighting coefficients are computed for first and second order by replacing M instead of N and y instead of x.

However the DQ interpolation needs the function value on the whole domain. so the solution domain must be regular. The governing equation of wave propagation descretization by DQ method obtains

$$\frac{\partial^2(\varphi)}{\partial x^2} + \frac{\partial^2(\varphi)}{\partial y^2} + k_0^2 n^2 \varphi = -\sum_{i=1}^{N_S} S_i \delta(x_{S_i}, z_{S_i}) \tag{14}$$

Finally the MSE descretization by DQ method is written as:

$$\sum_{k'=1}^{N} wx_{i,k'}^{(2)} \varphi_{k',j} + \sum_{k'=1}^{M} wy_{j,k'}^{(2)} \varphi_{i,k'} + k_0 n_{i,j}^2 \varphi_{i,j} = -\sum_{i=1}^{N_S} S_i \delta(x_{S_i}, z_{S_i}), \quad i = 1,2,\ldots,N, \quad j = 1,2,\ldots,M \tag{15}$$

4. Solution Method

By applying the discrete acoustic wave equation (equation 13 and 18) on all of the grid points in the problem domain. a large system of linear equations is obtained and it should be solved simultaneously over the whole domain.

$$[A]\{\varphi\} = \{b\} \tag{16}$$

where $[A]$ is an $(M \times N)^2$ complex matrix. and $\{b\}$ is an $(M \times N)$ vector. For solution of system of linear equations Conjugate Gradient (CG). the iterative scheme that proposed by Xu was applied to guarantee convergence of solving the linear system of equations[12]. Whereas the coefficient matrix $[A]$ is not diagonally-dominant. nor symmetric and positive definite. but for conjugate gradient iterative procedures matrix $[A]$ needs to be diagonally-dominant or symmetric and positive definite. so the following Gauss transformation that recommended by Panchang is applied to equation (16)[13].

$$[A^*][A]\{\varphi\} = [A^*]\{b\} \tag{17}$$

where $[A^*]$ is the complex conjugate transpose of $[A]$. Clearly $[A^*][A]$ is Hermitian and positive-definite matrix that is suitable for CG iterative scheme. The algorithm that is presented by Xu [12] is as follows:

Step 1. Select trial values $\{\varphi_0\}$ (i.e. 0^{th} iteration) for all nodes in the model domain where the solution is desired.

Step 2. Compute for all points $\{r_0\} = \{f\} - [A]\{\varphi_0\}$ and $\{p_0\} = [A^*]\{r_0\}$.

Step 3. Compute for the i^{th} iteration

$$\alpha_i = \frac{\| [A^*]\{r_i\} \|^2}{\| [A]\{p_i\} \|^2} \tag{18}$$

Step 4. Update $\{\varphi_{i+1}\} = \{\varphi_i\} + \alpha_i \{p_i\}$ for all points.

Step 5. Check for convergence of the solution. The criterion for convergence is

$$\frac{\| [A]\{\varphi_{i+1}\} - \{f\} \|^2}{\| \{\varphi_{i+1}\} \|^2} \prec \varepsilon \tag{19}$$

Where ε is a prescribed tolerance. If it is true. stop.
Step 6. Compute. for each grid point

$$\{r_{i+1}\} = \{r_i\} - \alpha_i [A]\{p_i\} \tag{20}$$

Step 7. Compute for the ith iteration

$$\beta_i = \frac{\| [A^*]\{r_{i+1}\} \|^2}{\| [A^*]\{r_i\} \|^2} \tag{21}$$

Step 8. Compute $\{p_{i+1}\} = [A^*]\{r_{i+1}\} + \beta_i \{p_i\}$.
Step 9. Set $i = i + 1$. and go to step 3.

5. Numerical Example (Model Verification)

In order to assess the computational efficiency and accuracy of LDQ method. two numerical examples are considered to validate the model. First example was acoustic wave propagation in rectangular domain which has an analytical solution and second was a wedge-shaped ocean appeared among the range-dependent benchmark solutions solicited by the Acoustical Society of America in 1987.

5.1. Example 1: Acoustic wave propagation in rectangular domain

This example has analytical solution and can be an appropriate case for validation of numerical modeling of the acoustic wave propagation with DQM. Numerical domain was showed in Fig. 3 in which $\Gamma 1$. $\Gamma 2$ and $\Gamma 3$ are rigid boundaries and $\Gamma 1$ is uniform potential boundary.

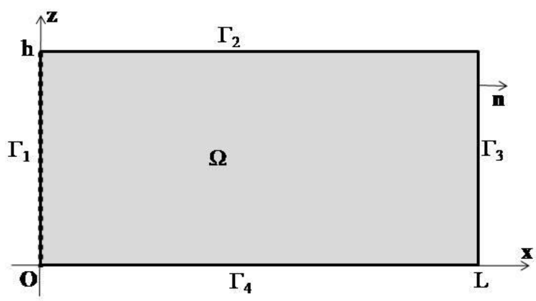


Figure 3. Example 1 domain geometry (L = 1 and h = 1)

$$\bar{\emptyset}(0.z) = \frac{1}{\omega\rho_0} cot(kL)\ for\ \{x = 0; 0 \le z \le L\} \tag{22}$$

$$\frac{\partial\bar{\emptyset}}{\partial n} = 0\ \ for \begin{Bmatrix} z = h; 0 \le x \le L \\ x = L; 0 \le z \le h \\ z = 0; 0 \le x \le L \end{Bmatrix} \tag{23}$$

where h is the domain height and L is the domain length. The problem has an exact solution given by.

$$\bar{\emptyset}(x.z) = \frac{1}{\omega\rho_0}\left\{sin(kx) + \frac{cos(kx)}{tan(kL)}\right\} \tag{24}$$

Fig. 4 shows the DQM solution of the acoustic wave propagation in the rectangular domain. Also. numerical result is compared with analytical solution in Fig 5. These figures show the accuracy of DQM numerical modeling by using small number of points for domain descritization.

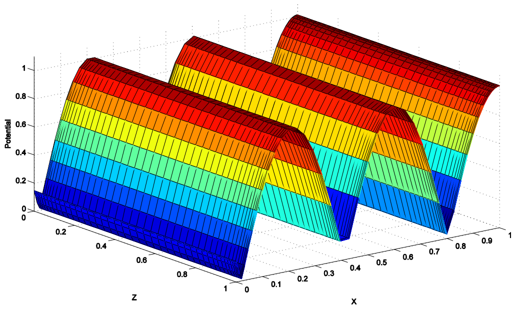


Figure 4. Numerical solution of acoustic wave equation in rectangular domain

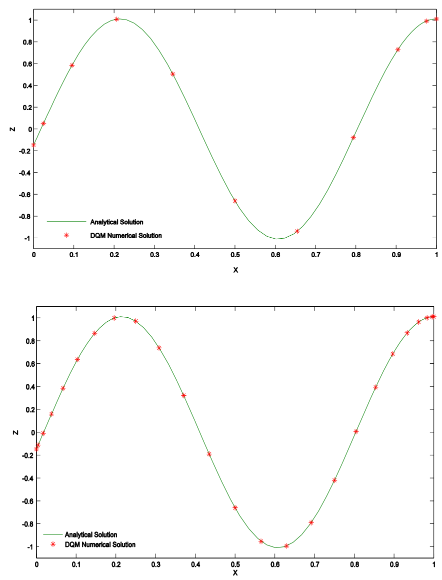


Figure 5. Comparison between DQM numerical results and analytical solutions

5.2. Example 2: Acoustic wave propagation in a wedge-shaped ocean

In this example applicability of DQM numerical modeling was studied in the more complex domain. The geometry domain was shown in Fig. 6. As shown in the figure a 25-Hz continuous wave source was located at zero range and mid depth in 200 m of water and the bottom slope was 1/20. Pressure receivers were located at depth of 30 and 150 m in horizontal arrays.

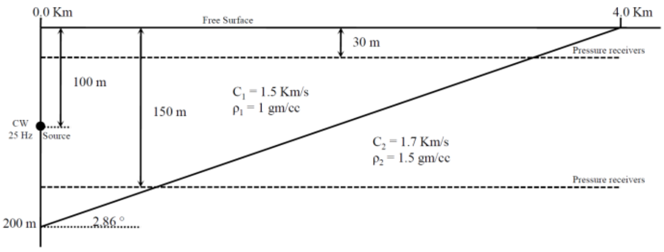
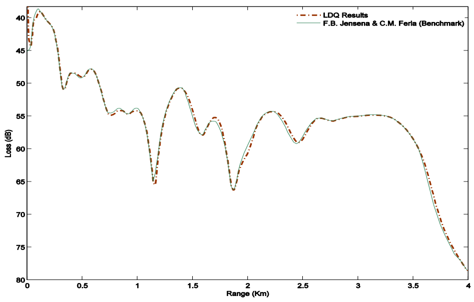
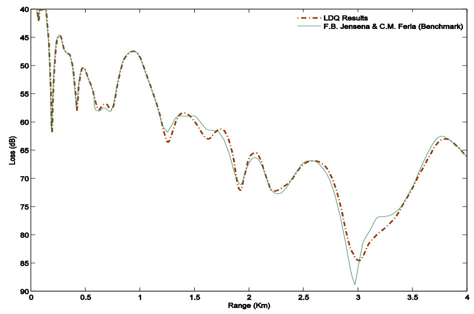


Figure 6. Acoustic wave propagation in a wedge-shaped ocean geometry

The solution for this problem appears in Fig. 7 which compares DQM numerical results with the F.B. Jensena & C.M. Ferla Benchmark data. The agreement is very good for the 30 m depth receivers and satisfactory for the 150 m receivers. These results demonstrated the accuracy and applicability of DQM numerical model for underwater acoustic wave propagation.



(a)



(b)

Figure 7. Numerical results of Acoustic wave propagation in a wedge-shaped ocean. a) for 30 m receivers b) for 150 m receivers

Well-known evaluation criteria were used to assess the performance of DQM compared to other numerical and experimental data.
The index of agreement is another usual evaluation criteria (Willmott. 1982).

$$d = 1 - \frac{\sum_{i=1}^{N_c} (P_i - O_i)^2}{\sum_{i=1}^{N_c} \left(|\tilde{P}_i| + |\tilde{O}_i|\right)^2} \tag{25}$$

Where (observation (O) and predicted (P)) $\tilde{P}_i = P_i - \overline{O}$. $\tilde{O}_i = O_i - \overline{O}$ and $\overline{O}$ is the mean of the observed data. The index of agreement varies between 0 and 1 where 0 shows the complete disagreement and 1 the perfect agreement.
Table 1 shows index of agreement above 0.98 for this example which certified that the numerical model can predict good results.

Table 1. Index of agreement for Acoustic wave propagation in a wedge-shaped ocean

Receiver Depth	d
30 m	0.9913
150 m	0.9801

6. Conclusions and discussion

Two numerical examples described in the paper certified that the DQ method accuracy and efficiency for solution of acoustic wave equation are appropriate. Even inasmuch as the DQ numerical gain better results rather than other numerical method in some sections by less grid point distribution in the domain.
Even in some sections the DQ model predicted better results than the other numerical results in contrast with analytical solution and benchmark example that was studied here. Note that these results were obtained for less grid point distribution in the computation domain and so the system of linear equations that was obtained by numerical discrete method can be solved by the direct method like gauss elimination.
These (PE) methods. despite their acceptable results for most range and depth dependent problems. are restricted to a small angle approximation and hence. may experience difficulties for long range problems.

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