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Recent Wave Breaking Prediction Formulas Evaluation Based On Compiled Laboratory Data

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ABSTRACT

One of the most important issues in the area of coastal structures design is determination of forces and loadings resulted from shallow water wave breaking. In the process of wave breaking, the subsequent particle motion is transformed from irrotational to rotational motion and due to this matter, vorticity and turbulence are generated and the sediment transport is affected by this phenomenon. Therefore, it is necessary to know about the location of wave breaking and other parameters such as the breaker height, breaker depth, etc. Over the last century, several formulas have been presented for predicting the wave breaking onset. These formulas depend on many parameters (e.g. seabed slope, water depth at the location of breaking, offshore wave height, etc.) that need to be known in order to obtain the desired wave breaking parameter (e.g. breaker height). In this study, some of the formulas for predicting wave breaking onset proposed in the recent decade are evaluated using the available laboratory data and it is tried to find out which formula is more suitable in different cases and conditions. A refinement process is carried out for choosing the appropriate data points out of all the available compiled laboratory data. The comparison is carried out in two phases. In the first one, the formulas are compared using all the data and in the second one, the comparisons are made based on the breaker type. These two phases yield to different outcomes. In the first phase, the formula proposed by Delavari et al. has the lowest values of bias, relative error, scatter index and root mean square error and the coefficient of determination of Goda's formula is the highest. In the second phase, the data are categorized based on the plunging and spilling breaker types and the comparisons are made based on this categorization. The outcomes derived from the first phase are different from the ones derived from the second one.

1. Introduction

Generally, as a wave propagates from deep water into shallow parts, its characteristics are changed and the wave steepness is increased due to shoaling. This increase is continued until the wave breaks at certain depth. Wave breaking also occurs in deep water, but the main focus here is on the wave breaking due to shoaling as a coastal process. When breaking is initiated, it develops quickly until the process is finished. In this process, a considerable amount of wave energy is released. For almost a century, many researchers have focused on the initiation of wave breaking because it's an important aspect in the design of coastal structures. The magnitude of wave forces and structures loadings depend on the wave height; thus it is necessary to predict the wave height in the instant of breaking (i.e. the breaker height). A considerable amount of formulas has been proposed by different researchers for predicting the wave height and water depth when wave breaking takes place [1]. Most of these formulas have two factors in common; the wave steepness and seabed slope. These two factors are almost the main factors for predicting the wave breaking initiation.

The aforementioned formulas can be classified into six different groups [1]. This classification is based on different factors that influence the four common indices for describing the breaker height. These indices are in the dimensionless form of H_b/d_b , H_b/H_o , H_b/L_b , and H_b/L_o . The subscripts *b* and *o* indicate the breaking and offshore conditions, respectively, and *H*, *d* and *L* are the wave height, water depth and wavelength, respectively. The first and second indices are known as the breaker depth index (γ_b) and the breaker height index (Ω_b), respectively. In this six-group categorization, the effective factors on the indices are the surf similarity parameter, breaking wavelength, offshore wavelength and height, and seabed slope. It should be mentioned that the surf similarity parameter ξ_o (also known as the Irribaren number) is used for categorizing the breaker type:

$$\xi_o = \frac{\tan\beta}{\sqrt{H_o/L_o}} \tag{1}$$

where β denotes the seafloor angle, H_o is the offshore wave height and L_o indicates the offshore wavelength. According to this parameter, the breaking type is classified into 3 groups [2] (i.e. surging/collapsing, plunging, and spilling).

The initial attempts to propose a new wave breaking predicting formula were based on simple laboratory tests with limited applications. For instance, McCowan studied wave breaking for the case of a solitary wave over a horizontal seafloor in 1894 [3] and proposed the formula $H_b = 0.78d_b$ for predicting breaker height. Thereafter, several other researches have been carried out with more complicated beach types and waves (including irregular and random waves) [1]. In 2010, Delavari et al. [4] proposed a formula using a statistical model based on the available laboratory data. The relation is given in the following:

$$H_b = L_0 \times \left[0.001m^{-0.358} + 0.608m^{-0.069}\left(\frac{H_0}{L_0}\right)^{0.768m^{-0.053}}\right] \tag{2}$$

in which, *m* is the seabed slope. As seen in Eq. (2), the breaker height can be calculated using the seabed slope and offshore wave height and wavelength. As stated by Delavari et al., the proposed relation is more accurate than other relations mentioned in Ref. [4] for predicting the breaker height.

Another formula investigated here is the formula proposed by Goda in 2010 [5]. The relation is as follows:

$$H_b = 0.17L_0\left[1-\exp\left(-1.5\frac{\pi d_b}{L_o}\left(1+11m^{\frac{4}{3}}\right)\right)\right] \tag{3}$$

Eq. (3) is a modification of Goda's initial wave breaking prediction formula (see Ref. [2] for more details). This modification has led to better agreement with the available laboratory data mentioned in Ref. [2].

In 2015, Robertson et al. [6] proposed a new wave breaking prediction formula using the artificial neural networks (ANNs). The new formula was evaluated by both laboratory and field data so that full-scale studies were done and the limitations of laboratory-based analyses (e.g. scaling issues, inability in scaling all characteristics of breaking, and etc.) were eliminated. The formula is given below:

$$H_b = 0.17L_0\left[1-\exp\left(\frac{\pi d_b}{L_o}\left(1.978m - 1.792\right)\right)\right] \tag{4}$$

The other formula investigated here is proposed by Antoniadis in 2018 [2]. This formula considers the effect of oblique waves on wave breaking. The relation is as follows:

$$H_b = 0.1657\cos\theta_0 - 0.1885m + 1.0284H_0 + 0.00189L_0 - 0.1504 \tag{5}$$

in which, θ_0 denotes the deep water wave angle. The accuracy of this formula in predicting the breaker height was shown to be better than other published formulas.

In this research, the wave breaking predicting formulas introduced in the previous paragraphs are compared and evaluated based on the available compiled laboratory data and it is tried to find the most accurate one. Note that some of the data points are ignored due to a refinement process described in the following section. The parameters such as seabed slope and offshore wavelength are common in all of the formulas investigated here. Two phases are considered in the comparison process. The first one contains all of the refined data points for comparing the formulas and in the second phase, the comparisons are based on the breaker type. As it is shown in the next parts, these phases lead to different outcomes.

2. Materials and methods

Twenty-five experimental datasets with overall 536 data points were gathered (as shown in Table 1). After carrying out a refinement process, about 370 data points were selected. They are used for comparing the aforementioned empirical relations with the available laboratory data.

The refinement process is based on two breaker type criteria described by Battjes [7] and Camenen and

Larson [8]. According to Battjes [7], the breaker type is described by the Irribaren number as below:

Table 1: The available compiled laboratory data

Dataset	$H_b\,(\mathrm{m})$	m	$T\,(\mathrm{s})$
Munk (1949)	0.068-0.099	0.009-0.072	0.86-1.98
Iversen (1952)	0.047-0.125	0.02-0.1	0.74-2.67
Morison and Crooke (1953)	0.055-0.113	0.02-0.1	0.78-2.62
Horikawa and Kuo (1967)	0.060-0.182	0.0125-0.05	1.2-2.3
Komar and Simmons (1968)	0.034-0.170	0.036-0.105	0.81-2.37
Galvin (1969)	0.038-0.177	0.05-0.2	1.0-6.0
Weggel (1970)	0.089-0.161	0.051	1.26-2.05
Saeki and Sasaki (1973)	0.099-0.106	0.02	1.3-2.5
Iwagaki et al. (1974)	0.044-0.128	0.03-0.1	1.0-2.0
Walker (1974)	0.024-0.115	0.033	1.17-2.33
Van Dorn (1978)	0.108-0.166	0.022-0.083	1.65-4.80
Hansen and Svendsen (1979)	0.071-0.102	0.0292	1.0-3.33
Singamsetti and Wind (1980)	0.073-0.193	0.025-0.2	1.03-1.73
Mizuguchi(1981)	0.1	0.1	1.2
Visser (1982)[1]	0.058-0.108	0.05-0.101	0.7-2.01
Stive (1986)	0.178	0.025	1.79
Smith and Kraus (1990)	0.082-0.164	0.033	1.02-2.49
Ting and Kirby (1994)	0.163-0.191	0.0286	2.0-5.0
Zou et al. (1999)	0.044-0.120	0.01-0.025	1.0-2.0
Hoque (2002)	0.117-0.207	0.1053	1.12-1.8
Deo and Jagdale (2003)	0.094-0.141	0.033-0.100	0.74-1.20
Cox and Shin (2003, 2006)	0.144-0.157	0.0286	1.5-3.0
Tsai et al. (2005)	0.135-0.305	0.1-0.333	1.8-2.6
Mori and Kakuno (2008)	0.120-0.165	0.033	1.6-3.8

$$\begin{cases} \text{spilling} & \text{if } \xi_0 \le 0.5 \\ \text{plunging} & \text{if } 0.5<\xi_0 \le 3.3 \\ \text{surging/collapsing} & \text{if } \xi_0 > 3.3 \end{cases} \tag{6}$$

In 2007, Camenen and Larson [8] proposed the following relationships between the Irribaren number and breaker depth for describing the breaker type:

$$\begin{cases} \text{spilling/plunging} \\ \quad \text{when } \xi_0 = 1.5 - 1.25\gamma_b \quad (\xi_0 = 0.2 - 0.6) \\ \text{plunging/collapsing} \\ \quad \text{when } \xi_0 = 4.4 - 2.5\gamma_b \quad (\xi_0 = 0.8 - 2.9) \\ \text{collapsing/surging} \\ \quad \text{when } \xi_0 = 1.7 + 3.3\gamma_b \quad (\xi_0 = 3.6 - 6) \end{cases} \tag{7}$$

If these two criteria yield to the same breaker type, then the data point will be used. Otherwise, the data is ignored.

As it is seen in Table 1, the ranges of observed breaker height, seabed slope and period are between 0.015-0.305m, 0.009-0.105 and 0.7-8.0 sec, respectively. It should be mentioned that in the experiment of Visser [9], different wave angles are used, i.e. oblique waves are considered.

In order to compare the observed breaker height (H_{bo}) and the calculated one (H_{bc}), the following statistical characteristics are used:

- Root mean square error (*RMSE*): A non-negative parameter used as a measure of accuracy. The model which has lower *RMSE* is more accurate.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}\left(H_{bc,i} - H_{bo,i}\right)^2}{N}} \tag{8}$$

- Coefficient of determination (R^2): A measure for showing how well a model explains and predicts future outcomes. The model with a value of R^2 closer to 1 is more fit and accurate.

$$R^2 = \left[\frac{\sum_{i=1}^{N}\left(H_{bc,i} - \bar{H}_{bc}\right)\left(H_{bo,i} - \bar{H}_{bo}\right)}{\sqrt{\sum_{i=1}^{N}\left(H_{bc,i} - \bar{H}_{bc}\right)^2}\sqrt{\sum_{i=1}^{N}\left(H_{bo,i} - \bar{H}_{bo}\right)^2}}\right]^2 \tag{9}$$

- Bias (*B*): A measure for overestimating or underestimating a parameter. The model that has smaller value of *B* is more accurate.

$$B = \frac{\sum_{i=1}^{N} H_{bc,i} - H_{bo,i}}{N} \tag{10}$$

- Percentage relative error (*E*) for average wave heights:

$$E(\%) = \left|\frac{\bar{H}_{bc} - \bar{H}_{bo}}{\bar{H}_{bc}}\right| \times 100 \tag{11}$$

- Scatter index (*SI*):

$$SI(\%) = \frac{RMSE}{\frac{1}{N}\sum_{i=1}^{N} H_{ob,i}} \times 100 \tag{12}$$

3. Results and discussion

The comparison process carried out in this study consists of two phases. In the first phase, the formulas (Eqs. (2-5)) are compared using all of the 370 refined data points and its outcomes are provided in Table 2. As seen in this table, the root mean square error (*RMSE*), relative error (*E*), bias (*B*) and scatter index (*SI*) of Delavari's formula are the lowest with the values of 0.0182m, 1.90%, 0.0021m, and 16.85%, respectively and Goda's formula has the highest coefficient of determination (R^2) with the value of 0.80. Therefore, it can be concluded that the formula proposed by Delavari has better and more accurate results and Goda's formula's results are less scattered than the others.

It should be mentioned that the formula proposed by Antoniadis considers the effect of wave direction but due to the lack of the data gathered here for oblique waves, all of the experiments are assumed to have head-on waves, except for Visser's experiment that includes oblique waves. Another thing that is worthwhile to mention here is that although the formula proposed by Robertson et al. [6] has about 4.5% error in predicting the breaker height, its main applicability is for full-scale field studies with the breaker height in the range of 0.8 and 3.5 meters, but here only the available laboratory data with the maximum of 0.3 m breaker height is considered.

Figure 1 illustrates the scatter diagrams of the results of comparison between the aforementioned formulas. As it can be seen, the formula proposed by Goda is a formula with very good statistical fits.

Another aspect considered here as the second phase of the comparison process is based on the breaker type. The available compiled refined laboratory dataset contains two types of breaker type; namely plunging and spilling. Tables 3 and 4 include the comparisons between the prediction formulas (Eqs. (2-5)) for spilling and plunging types, respectively.

It can be understood from Tables 3 and 4 that for the spilling breaker type, all formulas provide acceptable results. The formula proposed by Antoniadis has the lowest bias (B) and relative error (E) with the values of 0.0002m and 0.15%, respectively. The values of root mean square error ($RMSE$) and scatter index (SI) of Goda's formula are 0.0124m and 11.13%, respectively, which are the lowest values compared to the other formulas. This formula also has the highest coefficient of determination (R^2) with the value of 0.86. When the type of breaking changes to plunging, the values of root mean square error ($RMSE$), bias (B), relative error (E), and scatter index (SI) of the formula proposed by Delavari et al. are 0.0210m, 0.0021m, 2.01% and 20.66%, respectively and the coefficient of determination (R^2) of Antoniadis's formula is 0.87. Therefore, a formula that leads to the best results in the first phase does not necessarily lead to the same results when investigating different breaker types.

4. Conclusions

The wave breaking phenomena in shallow waters is considered as a substantial criterion in coastal engineering design and it is very important to know about the location and other characteristics of wave breaking so that the resulting forces and effects on coastal areas can be understood and estimated. For about 150 years, many researchers have been trying to propose new formulas for predicting wave breaking characteristics (e.g. breaker height). In this research, about 370 experimental cases from 25 different sources were used for evaluating four wave breaking prediction formulas proposed in the recent decade. The range of the seabed slope was between 0.009 and 0.105 and the observed breaker heights were between 0.015 and 0.307m. One of the formulas was proposed by Goda in 2010. Another one was the work of Delavari et al. in 2010. The other two formulas were the ones proposed by Robertson et al. in 2015 and Antoniadis in 2018. The evaluation of the formulas was carried out using a two-step comparison process. In the first step, all of the refined data points were used for the comparison and the in the second step, the data points with similar breaker type were categorized into groups and the comparison was based on the breaker type. It has been understood from the results that the formula proposed by Delavari et al. had less error and therefore more accuracy than the other formulas in the first step of comparison. Moreover, the value of the coefficient of determination of Goda's formula was higher than the other formulas. For the second step, when the breaker type was in the form of spilling, Antoniadis's formula had the lowest values of bias and relative error and the values of root mean square error and scatter index of Goda's formula was the lowest. This formula's coefficient of determination was also the highest. For the plunging breaker type, the formula proposed by Delavari et al. was more accurate than the others since its values of root mean square error, bias, relative error and scatter index was the lowest. For the coefficient of determination, the formula proposed by Antoniadis had the highest values. Since the number of refined data points used here is limited to 370, it is suggested to evaluate these formulas using a wider range of observed breaker heights to evaluate them and find out which one is more accurate when the conditions become closer to the full-scale field condition.

Table 2: Comparison between the calculated breaker height and the observed one

Formula	Average observed wave height $\bar{H}_{bo}$ (m)	Average calculated wave height $\bar{H}_{bc}$ (m)	$RMSE$ (m)	B (m)	R^2	E (%)	SI (%)
Delavari (2010)	0.1084	0.1105	0.0182	0.0021	0.79	1.90	16.85
Goda (2010)	0.1084	0.1121	0.0188	0.0038	0.80	3.36	17.31
Robertson (2015)	0.1084	0.1037	0.0216	-0.0046	0.72	4.46	19.94
Antoniadis (2018)	0.1084	0.1026	0.0202	-0.0058	0.76	5.63	16.69

Table 3: Comparison between the calculated breaker height and the observed one for spilling breaker type

Formula	Average observed wave height $\bar{H}_{bo}$ (m)	Average calculated wave height $\bar{H}_{bc}$ (m)	*RMSE* (m)	*B* (m)	R^2	*E* (%)	*SI* (%)
Delavari (2010)	0.1114	0.1141	0.0149	0.0027	0.81	2.39	13.41
Goda (2010)	0.1114	0.1121	0.0124	0.0008	0.86	0.67	11.13
Robertson (2015)	0.1114	0.1148	0.0168	0.0034	0.78	2.95	15.06
Antoniadis (2018)	0.1114	0.1116	0.0164	0.0002	0.78	0.15	14.74

Table 4: Comparison between the calculated breaker height and the observed one for plunging breaking type

Formula	Average observed wave height $\bar{H}_{bo}$ (m)	Average calculated wave height $\bar{H}_{bc}$ (m)	*RMSE* (m)	*B* (m)	R^2	*E* (%)	*SI* (%)
Delavari (2010)	0.1017	0.1038	0.0210	0.0021	0.81	2.01	20.66
Goda (2010)	0.1017	0.1115	0.0279	0.0098	0.78	8.81	27.48
Robertson (2015)	0.1017	0.0797	0.0293	-0.0220	0.86	27.6	28.84
Antoniadis (2018)	0.1017	0.0845	0.0242	-0.0172	0.87	20.4	23.80

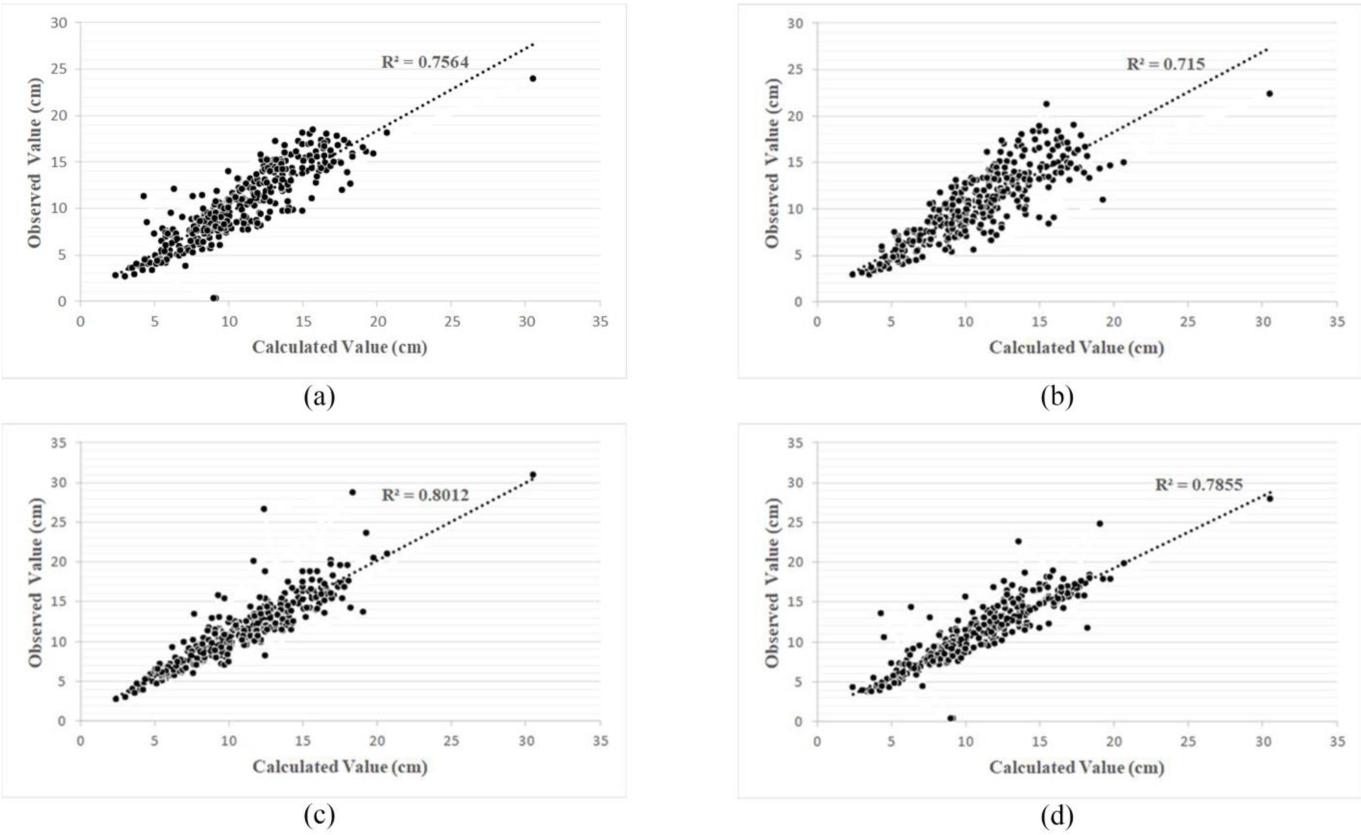

Figure 1. The scatter diagram of (a) Antoniadis's formula (b) Robertson's formula (c) Goda's formula (d) Delavari's formula

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6. References

1-Robertson, B., Hall, K., Zytner, R., and Nistor, I., (2013), *Breaking waves: Review of characteristic relationships*, Coastal Engineering Journal, Vol.55(1).

2-Antoniadis, C., (2018), *Experimental verification of wave breaking formulae for obliquely incident waves on mixed and gravel beaches*, International Journal of Oceanography & Aquaculture, Vol.2(1).

3-McCowan, J., (1894), *On the Highest Waves of a Permanent Type*, Philosophical Magazine, Edinburgh, Vol.38, p.351–358.

4-Delavari, E., Gharebaghi, M.A., and Chenaghloo, M.R., (2011), *Prediction of water wave breaking height and depth using ANFIS*, Proceedings of the ASME 2011 30th International Conference on Ocean, Offshore and Arctic Engineering, Rotterdam, The Netherlands.

5-Goda, Y., (2010), *Reanalysis of regular and random breaking wave statistics*, Coastal Engineering Journal, Vol.52, p. 71-106.

6-Robertson, B., Gharabaghi, B., and Hall, K., (2015), *Prediction of incipient breaking wave-heights using artificial neural networks and empirical relationships*, Coastal Engineering Journal, Vol.57.

7- Battjes, J., (1974), *Surf similarity*, Proceedings of the 14th International Conference on Coastal Engineering (Copenhagen, Denmark, ASCE), p. 1050–1061.
8-Camenen, B. and Larson, M., (2007), *Predictive formulas for breaker depth index and breaker type*, Journal of Coastal Research, Vol.23(4), p.1028–1041.
9-Visser, P.J., (1982) *The proper longshore current in a wave basin,* Laboratory of fluid mechanics, Delft University of Technology, Delft, The Netherlands.