

Comparison between Homotopy Analysis Method (HAM) and Variational Iteration Method (VIM) in Solving the Nonlinear Wave Propagation Equations in Shallow Water

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ABSTRACT

This study aims to investigate the capability of two common numerical methods, Homotopy Analysis Method (HAM) and Variational Iteration Method (VIM), and to suggest more efficient approximate solution method to the governing equations of nonlinear surface wave propagation in shallow water. To do so, semi-flat, moderate, and sharp slope of shore which are connected to an open ocean with a uniform depth are exposed to a solitary wave with initial wave height $H=2$ and stationary elevation $d=20$. Then, the surface elevation and velocity curves for these profiles are determined and compared by HAM and VIM. To verify the numerical modeling, two slopes i.e. semi-flat and moderate slope are considered and modeled in Flow-3D. Afterwards, the results of surface elevations are compared to each other by using correlation coefficient. The correlation coefficients for the slopes represent that the results coincide well. Ultimately, although the results of both methods are quite similar, using HAM is highly recommend rather than VIM since it makes solution procedure fast-converging and more abridged.

1. Introduction

Tsunamis are sea surface gravity waves generated by large-scale underwater disturbances. There are several stimuli that instigate these long waves: seismic displacement of seabed, volcanic eruptions, landslides, impact of large objects (such as astronomical objects) into the sea surface, and underwater explosions. As a result of these impulsive disturbances, water column—from the bottom to the free surface—is set in motion [1]. During the past era, waves like Tsunamis have imposed lots of irrevocable damages. Therefore, proposing numerical solutions for governing equations of waves has become a common practice in order to forecast waves more accurate [2-3]. For this purpose, several solution methods have been suggested, which Variational Iteration Method (VIM) and Homotopy Analysis Method (HAM) are almost the most common ones among them.

The concept of VIM stems from the studies performed by He in 1990s. According to these studies, VIM is an iterative based approach which does not need the presence of small parameters in the differential equation and is widely used for solution of nonlinear ordinary and partial differential equations. Therefore, this method has frequently been used and known as a reliable tool for solving linear and nonlinear wave

equations [4]. In 2006, Yusufglu et al applied VIM in order to regularized long wave equation [5]. In 2007, Hemeda used VIM for the wave equations in different forms i.e. first-order of wave equation in one- and two-dimension and second-order of wave equation in one- and two-dimension. Then, the results showed the effectiveness of this method [6]. In 2011, Mohyud-Din et al used the modified form of VIM to assess propagation of solitary wave by solving seventh order generalized Kdv (SOG-Kdv) equations [7]. In 2012, Younesian et al solved nonlinear wave propagation in shallow water media by VIM [8].

The ideas of HAM in topology was presented by Liao in 1992 in response to nonlinear wave problems. According to this study, it was claimed that the advantages of HAM outweighed other classical methods. The major advantage of this method is being independence of any small or large quantities. Therefore, HAM can be applied to governing equations and boundary/initial conditions containing whether small or large quantities. In addition, apart from providing more accurate and optimized solution without any physical and unrealistic assumptions, the numerical solution of HAM always becomes convergent since this method provides a family of solution expressions in the auxiliary parameter of $\hbar$

which makes the convergence region and rate of each solution convenient[9-16].

There are a lot of attempts made by means of HAM to solve waves equations with different aims. In 2004, Wu et al applied HAM to solve solitary waves governed by Camassa-Holm equation and provided a new analytical approach to solve soliton waves with discontinuity at crest [17]. In 2009, Yusufoglu et al performed HAM to solve Modified Equal Width Wave (MEW) equation and proposed this method as an efficient method for solving MEW equation [18]. In 2013, Shaiq et al recommended HAM as an accurate and reliable algorithm for time-fractional nonlinear wave-like equations which are significantly important in engineering issues [19]. In 2013, Araghi et al used HAM to solve Schrodinger equation with a power law nonlinearity [20]. In 2014, Izadian et al utilized a new approach with a fast and global quadratic rate of convergence for solving nonlinear wave equations by HAM and Newtown method [21]. In 2015, Yin et al used a modified form of HAM for solution of fractional wave equations and proposed this method as a powerful tool to adjust and control the convergence region of infinite series solution by using an auxiliary parameter [22].

According to the previous studies, it is claimed that both HAM and VIM could be used as efficient approaches in solving nonlinear equations. Therefore, this study aims to discuss the procedure of HAM approach and compare this method to VIM in order to investigate which method is more efficient. To do so, three different slopes: semi-flat, moderate, and sharp slope are firstly considered, and governing equations are solved by two algorithms. Secondly, the results of HAM are compared to the results of VIM approach. Then, the examples which are solved by both methods are verified by Flow-3D. Finally, although the responses obtained by HAM and VIM are similar, comparing the time lapse of both methods reveals that HAM reaches to convergent state with a lower computational load and never discretizes.

2. Governing Equations of nonlinear wave propagation

Tsunamis are generally classified as long waves. Solitary waves or combinations of negative and positive solitary-like waves are often used to simulate the run-up and shoreward inundation of these catastrophic waves.

The following equations display the specific case of the run-up of 2D long waves incident upon a uniform sloping beach connected to an open ocean with a uniform depth (Figure 1). The related classical nonlinear shallow-water equations are shown as Eq. (1):

$$\begin{aligned} \eta_t + (u(h + \eta))_x &= 0 \\ u_t + uu_x + g\eta_x &= 0 \end{aligned} \quad (1)$$

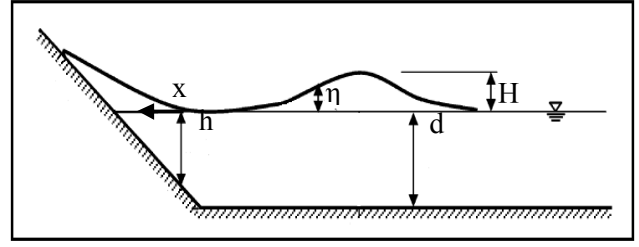


Figure 1. Definition Sketch for solitary wave run-up

where η is wave amplitude, u is depth averaged velocity, h is variable depth, and g is acceleration of gravity. In addition, the initial condition of these wave is generally represented by Eq. (2):

$$\begin{aligned} \eta(x, 0) &= H \operatorname{sech}^2 \sqrt{\frac{3H}{4d^3}} x \\ u(x, 0) &= \frac{\eta}{d} \sqrt{gd} \end{aligned} \quad (2)$$

where H and d denote the initial wave height and stationary elevation, respectively [1,8].

2.1. The basic idea of Homotopy Analysis Method (HAM)

To show the basic idea of HAM, the following procedure is considered. At first, differential equation is considered as Eq. (3):

$$\mathcal{N}[\omega(x, t)] = 0 \quad (3)$$

Where $\mathcal{N}$ is a nonlinear operator, x and t represent the independent variables, and ω is an unknown function. Then, all boundaries or initial conditions are ignored for simplicity, and the deformation equation which is so-called zeroth-order deformation equation is constructed Eq. (4).

$$(1-q)\mathcal{L}[\phi(x, t; q) - \omega_0(x, t)] = q\hbar\mathcal{N}[\phi(x, t; q)] \quad (4)$$

Where $q \in [0, 1]$ is the embedding parameter, $\hbar \neq 0$ is an auxiliary parameter, $\mathcal{L}$ is an auxiliary linear operator, $\phi(x, t; q)$ is an unknown function, $\omega_0(x, t)$ is an initial guess of $\omega(x, t)$, and $\phi(x, t; q)$ is an unknown function. It is obvious when q , the embedding parameter, is equal to 0 and 1, Eq. (4) becomes Eqs. (5):

$$\begin{aligned} \phi(x, t; 0) &= \omega_0(x, t) \\ \phi(x, t; 1) &= \omega(x, t) \end{aligned} \quad (5)$$

respectively. Thus, as q increases from 0 to 1, the solution varies from the initial guess $\omega_0(x, t)$ to the solution $\omega(x, t)$. Expanding $\phi(x, t; q)$ in Taylor series when q is equal to 1 will be Eq. (6):

$$\phi(x, t; q) = \omega_0(x, t) + \sum_{m=1}^{\infty} \omega_m(x, t) q^m \quad (6)$$

Where

$$\omega_m(x, t) = \frac{1}{m!} \left. \frac{\partial^m \phi(x, t; q)}{\partial q^m} \right|_{q=0} \quad (7)$$

The convergence of the series in Eq. (6) depends upon the auxiliary parameter $\hbar$. If it is convergent at $q = 1$, it will be Eq. (8):

$$\omega(x, t) = \omega_0(x, t) + \sum_{m=1}^{\infty} \omega_m(x, t) \quad (8)$$

Which must be one of the solutions of the original nonlinear equation, as proven by Liao. Then, $\overline{\omega_n}$ is defined as Eq. (9):

$$\overline{\omega_n} = (\omega_0(x, t), \omega_1(x, t), \dots, \omega_n(x, t)) \quad (9)$$

Therefore, the following m th-order deformation equation is obtained by differentiating Eq. (3) m -times with respect to q , dividing them by $m!$, and finally setting $q = 0$. Then, we have Eq. (10):

$$\mathcal{L}[\omega_m(x, t) - \chi_m \omega_{m-1}(x, t)] = \hbar R_m(\overline{\omega_{m-1}}) \quad (10)$$

Where

$$R_m(\overline{\omega_{m-1}}) = \frac{1}{m!} \left. \frac{\partial^{m-1} N[\phi(x, t; q)]}{\partial q^{m-1}} \right|_{q=0} \quad (11)$$

And

$$\chi_m = \begin{cases} 0, & m \leq 1 \\ 1, & m > 1 \end{cases} \quad (12)$$

It should be emphasized that $\omega_m(x, t)$ for $m \geq 1$ is governed by the linear equation of Eq. (10) with linear boundary conditions coming from the original problem, which can be solved by the symbolic computation software such as Mathematica or Maple. To perform HAM, the following initial approximations Eqs. (13) are considered:

$$\eta_0(x, 0) = \eta(x, 0) = H \operatorname{sech}^2 \sqrt{\frac{3H}{4d^3}} x \quad (13)$$

$$u_0(x, 0) = u(x, 0) = \frac{\eta}{d} \sqrt{gd}$$

And the linear operator is defined as Eq. (14):

$$\mathcal{L}_i[\phi_i(x, t; q)] = \frac{\partial \phi_i(x, t; q)}{\partial t}, i = 1, 2 \quad (14)$$

According to Eq. (1) nonlinear operators $\mathcal{N}_1$ and $\mathcal{N}_2$ can be defined as Eqs. (15):

$$\mathcal{N}_1[\phi_1(x, t; q), \phi_2(x, t; q)] = \frac{\partial \phi_1(x, t; q)}{\partial t} + \frac{\partial(\phi_1(x, t; q)\phi_2(x, t; q))}{\partial x} + \frac{\partial(h(x)\phi_2(x, t; q))}{\partial x} \quad (15)$$

$$\mathcal{N}_2[\phi_1(x, t; q), \phi_2(x, t; q)] = \frac{\partial \phi_2(x, t; q)}{\partial t} + \phi_2 \frac{\partial \phi_2(x, t; q)}{\partial x} + g \frac{\partial \phi_1(x, t; q)}{\partial x}$$

Then, zeroth-order deformation equations are constructed as Eqs. (16):

$$\begin{aligned} (1-q)\mathcal{L}_1[\phi_1(x, t; q) - \eta_0(x, t)] &= q\hbar_1 \mathcal{N}_1[\phi_1(x, t; q), \phi_2(x, t; q)] \\ (1-q)\mathcal{L}_2[\phi_2(x, t; q) - u_0(x, t)] &= q\hbar_2 \mathcal{N}_2[\phi_1(x, t; q), \phi_2(x, t; q)] \end{aligned} \quad (16)$$

Obviously, when $q = 0$ and $q = 1$, we have Eqs. (17):

$$\begin{aligned} \phi_1(x, t; 0) &= \eta_0(x, t), \quad \phi_1(x, t; 1) = \eta(x, t) \\ \phi_2(x, t; 0) &= u_0(x, t), \quad \phi_2(x, t; 1) = u(x, t) \end{aligned} \quad (17)$$

Thus, as the embedding parameter q increases from 0 to 1, $\phi_1(x, t; q)$ and $\phi_2(x, t; q)$ vary from the initial approximations of $\eta_0(x, t)$ and $u_0(x, t)$ to $\eta(x, t)$ and $u(x, t)$ solutions, respectively. By expanding $\phi_1(x, t; q)$ and $\phi_2(x, t; q)$ in Taylor series with respect to q , we have Eqs. (18) and Eqs. (19):

$$\phi_1(x, t; q) = \eta_0(x, t) + \sum_{m=1}^{\infty} \eta_m(x, t) q^m \quad (18)$$

$$\phi_2(x, t; q) = u_0(x, t) + \sum_{m=1}^{\infty} u_m(x, t) q^m$$

Where

$$\eta_m(x, t) = \frac{1}{m!} \left. \frac{\partial^m \phi_1(x, t; q)}{\partial q^m} \right|_{q=0} \quad (19)$$

$$u_m(x, t) = \frac{1}{m!} \left. \frac{\partial^m \phi_2(x, t; q)}{\partial q^m} \right|_{q=0}$$

If the auxiliary linear operator, the initial approximations, and the auxiliary parameters $\hbar_1$ and $\hbar_2$ are so properly chosen, the above series converge at $q = 1$. Then, we have Eqs. (20):

$$\eta(x, t; q) = \eta_0(x, t) + \sum_{m=1}^{\infty} \eta_m(x, t) \quad (20)$$

$$u(x, t; q) = u_0(x, t) + \sum_{m=1}^{\infty} u_m(x, t)$$

Which must be one of solutions of original system. Afterwards, the following vectors Eqs. (21) are defined:

$$\begin{aligned}\overline{\eta_n} &= (\eta_0(x, t), \eta_1(x, t), \dots, \eta_n(x, t)) \\ \overline{u_n} &= (u_0(x, t), u_1(x, t), \dots, u_n(x, t))\end{aligned}\quad (21)$$

And the m^{th} -order deformation equation, Eq. (22), is obtained:

$$\mathcal{L}_1[\eta_m(x, t) - \chi_m \eta_{m-1}(x, t)] = \hbar_1 R_{1,m}(\overline{\eta_{m-1}}, \overline{u_{m-1}}) \quad (22)$$

Now, the solution of the m^{th} -order deformation equation, Eq. (22), for $m \geq 1$ is Eqs. (23):

$$\begin{aligned}\eta_m(x, t) &= \chi_m \eta_{m-1}(x, t) + \hbar_1 \int_0^t R_{1,m}(\overline{\eta_{m-1}}, \overline{u_{m-1}}) ds \\ u_m(x, t) &= \chi_m u_{m-1}(x, t) + \hbar_2 \int_0^t R_{2,m}(\overline{\eta_{m-1}}, \overline{u_{m-1}}) ds\end{aligned}\quad (23)$$

According to the initial conditions which are initially assumed, we have Eq. (24):

$$\eta_m(x, 0) = 0, \quad u_m(x, 0) = 0 \quad (24)$$

$\mathcal{R}_{1,m}(\overline{\eta_{m-1}}, \overline{u_{m-1}})$ and $\int_0^t \mathcal{R}_{2,m}(\overline{\eta_{m-1}}, \overline{u_{m-1}})$ are:

$$\begin{aligned}\mathcal{R}_{1,m}(\overline{\eta_{m-1}}, \overline{u_{m-1}}) &= \frac{\partial \eta_{m-1}}{\partial t} + \frac{\partial}{\partial x} \left(\sum_{n=0}^{m-1} \eta_n u_{m-1-n} \right) \\ &+ h(x) \frac{\partial}{\partial x} u_{m-1} \\ \mathcal{R}_{2,m}(\overline{\eta_{m-1}}, \overline{u_{m-1}}) &= \frac{\partial u_{m-1}}{\partial t} + \left(\sum_{n=0}^{m-1} u_n \frac{\partial u_{m-1-n}}{\partial x} \right) + g \frac{\partial}{\partial x} \eta_{m-1}.\end{aligned}\quad (25)$$

Obviously, the solution of the m^{th} -order deformation equations, Eq. (22), for $m \geq 1$ becomes:

$$\begin{aligned}\eta_m &= \chi_m \eta_{m-1} + \hbar_1 \mathcal{L}^{-1}[\mathcal{R}_{1,m}(\overline{\eta_{m-1}}, \overline{u_{m-1}})] \\ u_m &= \chi_m u_{m-1} + \hbar_2 \mathcal{L}^{-1}[\mathcal{R}_{2,m}(\overline{\eta_{m-1}}, \overline{u_{m-1}})]\end{aligned}\quad (26)$$

To make the solution method more simple, h_1 and h_2 are assumed equal to h .

2.2. The basic idea of Variational Iteration Method (VIM)

To show the procedure of VIM, the correction functional is constructed at first as Eq. (27) and Eq. (28) [8]:

$$u_{n+1}(t) = u_n(t) - \int_0^t \left(u_n + E_n(\tau) \right) d\tau \quad (27)$$

$$\eta_{n+1}(t) = \eta_n(t) - \int_0^t \left(\eta_n + F_n(\tau) \right) d\tau \quad (28)$$

Where E_n and F_n are :

$$E_n = u_n u_n' + g \eta_n' \quad (29)$$

$$F_n = (u_n (\eta_n + H))' \quad (30)$$

The corresponding first-order iterations are obtained by Eq. (31) and Eq. (32):

$$u_1 = u_0(t) - \int_0^t \left(u_n + u_0 u_0' + g \eta_0' \right) d\tau = u_0 - (u_0 u_0' + g \eta_0') t \quad (31)$$

$$\eta_1 = \eta_0(t) - \int_0^t \left(\eta_0 + (u_0 (\eta_0 + H))' \right) d\tau = \eta_0 - (u_0 (\eta_0 + H))' t \quad (32)$$

The second-order approximations are:

$$\begin{aligned}u_2 &= u_0 - (u_0 u_0' + g \eta_0') - \int_0^t \left(u_1 + u_1 u_1' + g \eta_1' \right) d\tau = \\ &u_0 + t[-(u_0 u_0' + g \eta_0')] + \frac{t^2}{2} [u_0 (u_0 u_0' + g \eta_0') + u_0' \times \\ &(u_0 u_0' + g \eta_0') + g (u_0 (\eta_0 + H))''] + \frac{t^3}{3} \times \\ &[-(u_0 u_0' + g \eta_0') (u_0 u_0' + g \eta_0')] \quad (33)\end{aligned}$$

$$\begin{aligned}\eta_2 &= \eta_0 - (u_0 (\eta_0 + H))' t - \int_0^t \left(\eta_1 + (u_1 (\eta_1 + H))' \right) d\tau = \eta_0 + t[-u_0' (\eta_0 + H) - \\ &u_0 (\eta_0' + H')] + \frac{t^2}{2} [u_0' (u_0 \eta_0 + H))' + (\eta_0 + H) (u_0 u_0' + g \eta_0')] \\ &+ u_0 (u_0 (\eta_0 + H))'' + (\eta_0' + H') (u_0 u_0' + g \eta_0')] \\ &+ \frac{t^3}{3} [- (u_0 u_0' + g \eta_0') (u_0 (\eta_0' + H'))' - (u_0 u_0' + g \eta_0') (u_0 \eta_0 + H))''] \quad (34)\end{aligned}$$

Then surface elevation and velocity profiles are obtained after substituting the general initial conditions (Eq. (2)) into Eq. (35) and Eq. (36).

$$\begin{aligned}u_2 &= B \operatorname{sech}^2 \alpha x + 2t \times (B^2 \alpha \operatorname{sech}^5(\alpha x) \sinh^2(\alpha x) \\ &- g A \alpha \sinh \alpha \operatorname{sech}^3(\alpha x) \times \frac{t^2}{2} - 2B^3 \alpha^2 \operatorname{sech}^8(\alpha x) \cosh^2(\alpha x) \\ &+ 22gAB \alpha^2 \sec \alpha x \sin(\alpha x) - 6gAB \alpha^2 \operatorname{sech}^6(\alpha x) \\ &\cosh^2(\alpha x) - 2gB \alpha^2 H \cosh^2(\alpha x) \operatorname{sech}^4(\alpha x) + 6gB \alpha^2 H \\ &\operatorname{sech}^4(\alpha x) \times \sinh^2(\alpha x) \times BgH'' \operatorname{sech}^2(\alpha x) + 8g \alpha^2 \\ &AB \sinh^2(\alpha x) + \operatorname{sech}^6(\alpha x) - 4\alpha BH' \sinh(\alpha x) \operatorname{sech}^3(\alpha x) \\ &+ \frac{t^3}{3} [-8\alpha^3 B^4 \sinh^3(\alpha x) \operatorname{sech}^{11}(\alpha x) - 8g \alpha^3 AB^2 \sinh^2(\alpha x) \\ &\operatorname{sech}^9(\alpha x) + 4B^4 \alpha^4 \operatorname{sech}^9(\alpha x) \sinh(\alpha x) - 12B^4 \alpha^4 \\ &\operatorname{sech}^{11}(\alpha x) \sinh^2(\alpha x) + 8g \alpha^3 AB^2 \operatorname{sech}^7(\alpha x) \sinh(\alpha x) - \\ &24g \alpha^3 AB \operatorname{sech}^9(\alpha x) \sinh^2(\alpha x) + 4Ag^2 \alpha^3 \\ &\operatorname{sech}^7 \alpha x \sinh(\alpha x) - 12g^2 \alpha^3 A^2 \operatorname{sech}^5(\alpha x) \sinh(\alpha x)] \quad (35)\end{aligned}$$

And

$$\begin{aligned} \eta_2 = & A \operatorname{sech}^2(\alpha x) + t \times [-4\alpha AB \sinh(\alpha x) \operatorname{sech}^5(\alpha x) \\ & - 2\alpha ABH' \sinh(\alpha x) \operatorname{sech}^3(\alpha x) + BH' \operatorname{sech}^2(\alpha x) + t^2 \times \\ & [(-8\alpha^2 AB^2 - 16\alpha^2 AB^2 \alpha^2 - 12AB^2 \alpha^2 - 6\alpha^2 AB^2) \\ & \operatorname{sech}^8(\alpha x) + (36\alpha^2 AB^2 - 20\alpha^2 B^2 g - 10\alpha^2 A^2 g) \\ & \operatorname{sech}^6(\alpha x) + (16\alpha^2 B^2 - 6Ag\alpha)H \operatorname{sech}^4(\alpha x) + \\ & (-20\alpha^2 B^2 H') \operatorname{sech}^6(\alpha x) - 8\alpha B^2 H' \operatorname{sech}^5(\alpha x) \sinh(\alpha x) \\ & - 2A\alpha gH' \operatorname{sech}^3(\alpha x) \sinh(\alpha x) + 4HA\alpha^2 g \operatorname{sech}^2(\alpha x)] \end{aligned} \quad (36)$$

3. Numerical experiments

Now that the procedures of HAM and VIM have been discussed, three different examples with different profile shores (i.e. semi-flat, moderate, and sharp slopes) are modeled as follow by aforementioned approaches. Then, the following examples are exposed to a solitary wave with initial wave height $H = 2$ and stationary elevation $d = 20$ [8]. Then, the results which are determined by HAM are compared to those obtained by VIM [23].

3.1. Semi-Flat Shores

For modeling a semi-flat shore, the below shore profile is firstly considered Eq. (37):

$$h(x) = 0.2x - 20 \quad (37)$$

Then, the elevation profile Eq. (40) and surface elevation Eq. (41) are obtained by the following analytical expressions for HAM after initial amounts of $\eta_0(x, t)$ and $u_0(x, t)$ are determined by Eq. (38) and Eq. (39):

$$\eta_0(x, t) = 2 \operatorname{sech}[0.0137x]^2 \quad (38)$$

$$u_0(x, t) = 1.4007 \operatorname{sech}[0.0137x]^2 \quad (39)$$

$$\begin{aligned} \eta_1(x, t) = & -0.1534t (-1.8257h \operatorname{sech}^2[0.0137x] - \\ & 5h \operatorname{sech}^2[0.0136x] \tanh[0.0137x] + 0.05x h \\ & \operatorname{sech}^2[0.0137x] \tanh[0.0137x] + h \operatorname{sech}^4[0.0137x] \\ & \tanh[0.0137x]) \end{aligned} \quad (40)$$

$$\begin{aligned} u_1(x, t) = & -0.0537t (10h \operatorname{sech}^2[0.0137x] \\ & \tanh[0.0137x] + h \operatorname{sech}^4[0.0137x] \tanh[0.0137x]) \end{aligned} \quad (41)$$

The rest of the components of the iteration formulas by HAM can easily be obtained by symbolic computation software. Subsequently, the following approximate

solutions in term of a series up to 4th-order for $\eta = \sum_{i=0}^4 \eta_i$

and $u = \sum_{i=0}^4 u_i$ are obtained. The series solutions

contain the auxiliary parameter $\hbar$. The validity of the

method is based on assumption that the series of Eq. (6) converges at $q = 1$. In a essence, It is the auxiliary parameter $\hbar$ which ensures that this assumption can be satisfied. As it was pointed out by Liao, in general, by means of the so-called $\hbar$ -curve in Figures 2 and 3, it is straightforward to choose a proper value of $\hbar$ which results in convergence of the series. In addition, Liao mentioned that the valid region of $\hbar$ is a horizontal line segment [13]. So, $\hbar = -1$ is chosen in following computational works.

After the series become converged in HAM, the equations are solved on the bases of VIM approach as follows, and the results are shown in figures 4-7.

The solutions of VIM for $u_2(x, t)$ and $\eta_2(x, t)$ can be obtained by the following analytical expressions Eq. (42) and Eq. (43):

$$\begin{aligned} u_2 = & 1.400714 \operatorname{sech}^2(0.013693x) + 2t \times \\ & (0.2686566 \sinh(0.013693x) \operatorname{sech}^3(0.013693x) \\ & + 0.02686566 \sinh(0.013693x) \cosh^5(0.013693x)) \\ & \times t^2 (-0.00367014 \operatorname{sech}^8(0.013693x) - 0.07407 \end{aligned} \quad (42)$$

$$\begin{aligned} \eta_2 = & 2 \operatorname{sech}^2(0.013693x) + t[0.15336 \times \\ & \sinh(0.013693x) \sinh^5(0.013693x) - 0.280143 + \\ & \operatorname{sech}^2(0.013693x) 0.007672x \sinh(0.013693x) \\ & \sinh^3(0.013693x) - 0.7672 \sinh(0.013693x) \\ & - 0.05373 \operatorname{sech}^3(0.013693x) \sinh(0.013693x) \\ & - 0.021492 \operatorname{sech}^5(0.013693x) \sinh(0.013693x) \\ & - 0.00007375x \operatorname{sech}^6(0.013693x) - 0.0016186x \\ & \operatorname{sech}^4(0.013693x)] \end{aligned} \quad (43)$$

In Figures 4 and 5, graphical solutions of $\eta(x, t)$ and $u(x, t)$ are represented. The elevation and velocity profiles are illustrated versus time and position in Figures 6 and 7. By comparing these figures, it is concluded that HAM and VIM solutions are exactly similar.

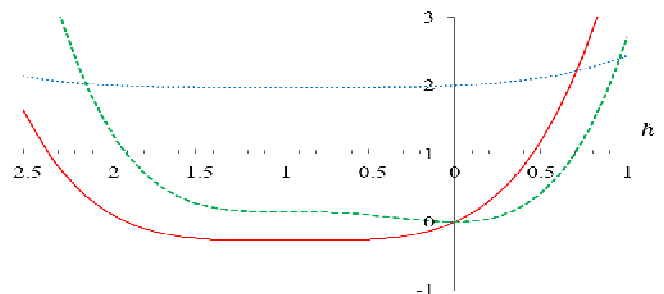


Figure 2. The $\hbar$ -curves according to 4th-order approximation. Dashed point: $\eta(0.1, 0.1)$, solid line: $\eta(0.1, 0.1)$, and dashed line: $\eta(0.1, 0.1)$.

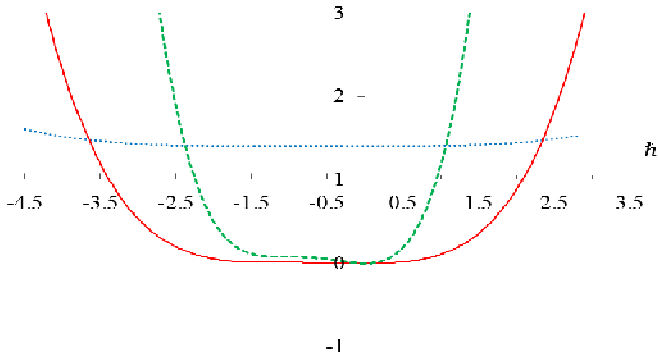


Figure 3. The h -curves according to 4th-order approximation. Dashed point: $u(0.1, 0.1)$, solid line: $\hat{u}(0.1, 0.1)$, and dashed line: $\tilde{u}(0.1, 0.1)$.

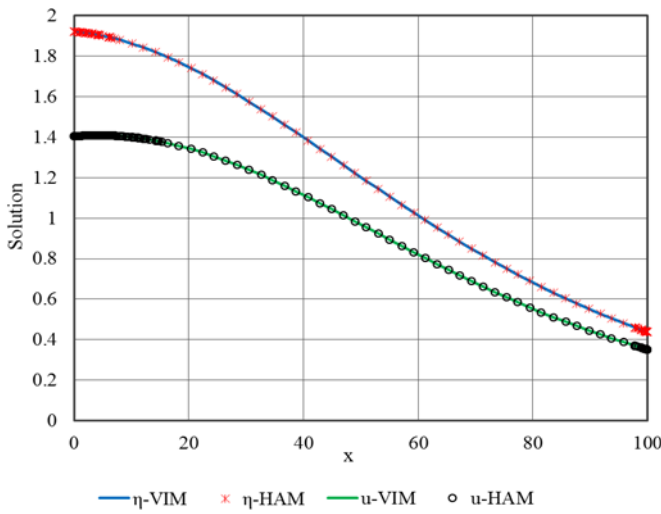


Figure 4. HAM and VIM solutions of $\eta(x, t)$ and $u(x, t)$ for semi-flat shore, $t = 0.3$ and $0 \leq x \leq 100$

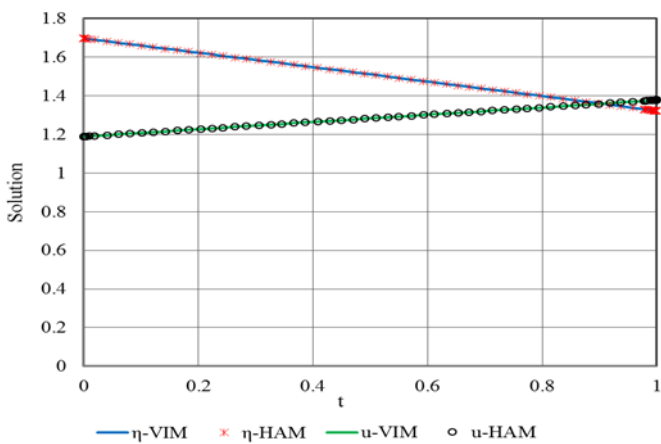


Figure 5. HAM and VIM solutions of $\eta(x, t)$ and $u(x, t)$ for semi-flat shore, $x = 30$ and $0 \leq t \leq 1$.

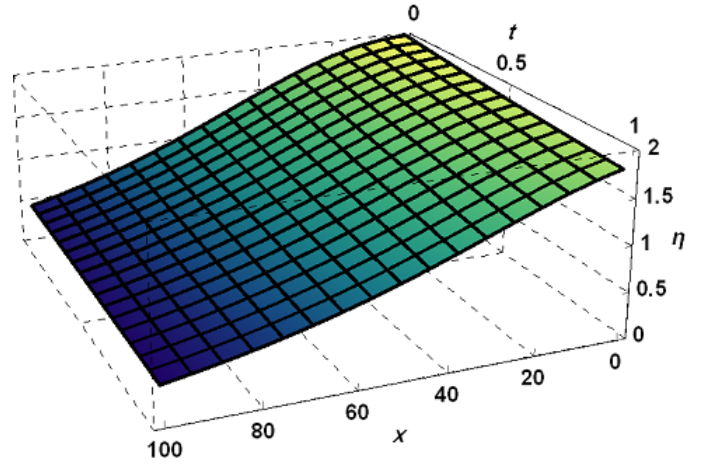


Figure 6. HAM solution of $\eta(x, t)$ for semi-flat shore

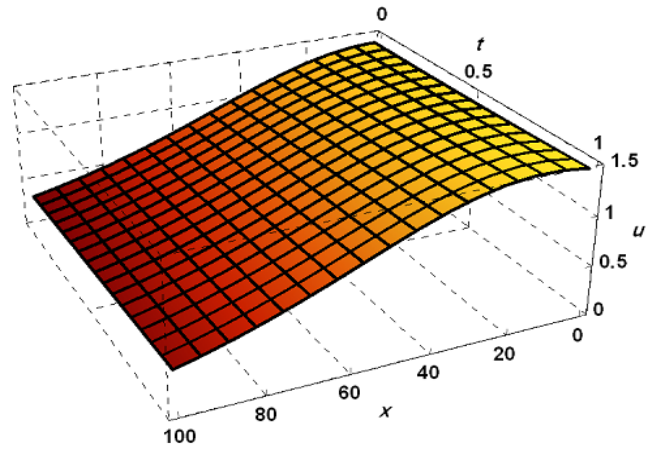


Figure 7. HAM solution of $u(x, t)$ for semi-flat shore

3.2. Moderate-Slope Shores

for modeling a moderate-slope shore, the below shore Eq. (44) profile is firstly considered:

$$h(x) = x - 100 \quad (44)$$

Then, the elevation profile Eq. (47) and surface elevation Eq. (48) are obtained by the following analytical expressions for HAM after initial amounts of $\eta_0(x, t)$ and $u_0(x, t)$ are determined (Eq. (45) and Eq. (46)):

$$\eta_0(x, t) = 2 \operatorname{sech}^2[0.0137x] \quad (45)$$

$$u_0(x, t) = 1.4007 \operatorname{sech}[0.0137x] \quad (46)$$

$$\begin{aligned} \eta_1(x, t) = & -0.1534 t \times (-9.12871 \hbar \operatorname{sech}^2[0.0137x] \\ & - 25 \hbar \operatorname{sech}^2[0.0137x] \tanh[0.0137x] \\ & + 0.25 x \hbar \operatorname{sech}^2[0.0137x] \tanh[0.0137x] \\ & + \hbar \operatorname{sech}^4[0.0137x] \tanh[0.0137x]) \end{aligned} \quad (47)$$

$$\begin{aligned} u_1(x, t) = & -0.0537 t (10 \hbar \operatorname{sech}^2[0.0137x] \\ & \tanh[0.0137x] + \hbar \operatorname{sech}^4[0.0137x] \\ & \tanh[0.0137x]) \end{aligned} \quad (48)$$

The auxiliary parameter of $\hbar$ is considered equal to -1, and the series will be expanded up to 4th-order. After the series become converged in HAM, the equations are solved on the bases of VIM approach by the following analytical expressions Eq. (49) and Eq. (50):

$$\begin{aligned} u_2 = & 1.400714 \operatorname{sech}^2(0.013693x) + 2t \\ & (0.2686566 \sinh(0.013693x) \operatorname{sech}^3(0.013693x) \\ & + 0.02686566 \sinh(0.013693x) \cosh^5(0.013693x)) \\ & \times t^2 (-0.00367014 \operatorname{sech}^8(0.013693x) \\ & \operatorname{sech}^4(0.013693x) - 0.515287 \operatorname{sech}^2(0.013693x) \\ & - 0.3766313 \operatorname{sech}^3(0.013693x) \sinh(0.013693x) \\ & + 0.00515288x \operatorname{sech}^2(0.013693x) - 0.007729x \times \\ & \operatorname{sech}^4(0.013693x) \end{aligned} \quad (49)$$

$$\begin{aligned} \eta_2 = & 2 \operatorname{sech}^2(0.013693x) + t(0.15336 \\ & \sinh(0.013693x) \operatorname{sech}^5(0.013693x) \\ & - 1.4007 \operatorname{sech}^2(0.013693x) - 3.836 \\ & \sinh(0.013693x) \operatorname{sech}^3(0.013693x) + \\ & 0.34433 \operatorname{sech}^6(0.013693x) + 0.83875 \\ & \operatorname{sech}^4(0.013693x) - 0.735 \operatorname{sech}^2(0.013693x) \\ & + 0.00735x \operatorname{sech}^2(0.013693x) - 0.26865 \\ & \operatorname{sech}^3(0.013693x) \sinh(0.013693x) \\ & - 0.00367875x \operatorname{sech}^6(0.013693x) - 0.008093x \\ & \operatorname{sech}^4(0.013693x) \end{aligned} \quad (50)$$

Then, the results are shown in figures 8-11. In Figures 8 and 9, the graphical solutions of $\eta(x, t)$ and $u(x, t)$ are represented. And also, the elevation and velocity profiles are illustrated versus time and position in Figures 10 and 11. By comparing these figures, it is concluded that HAM and VIM solutions are exactly similar.

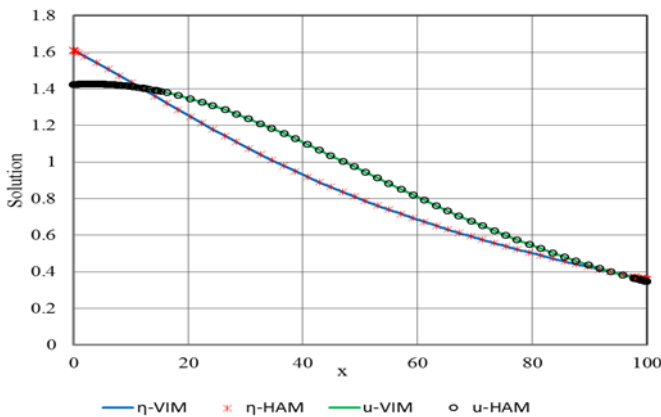


Figure 8. HAM and VIM solutions of $\eta(x, t)$ and $u(x, t)$ for moderate-slope shore at $t = 0.3$ in $0 \leq x \leq 100$.

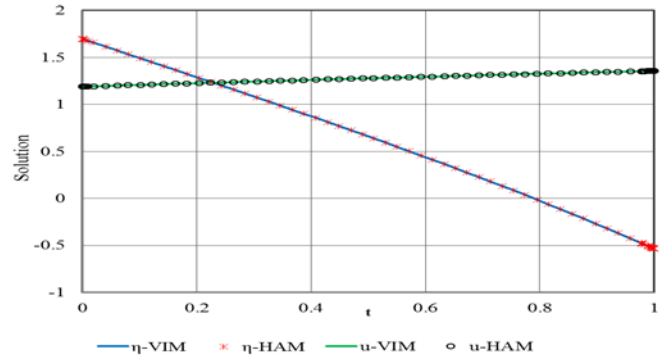


Figure 9. HAM and VIM solutions of $\eta(x, t)$ and $u(x, t)$ for moderate-slope shore at in $0 \leq t \leq 1$.

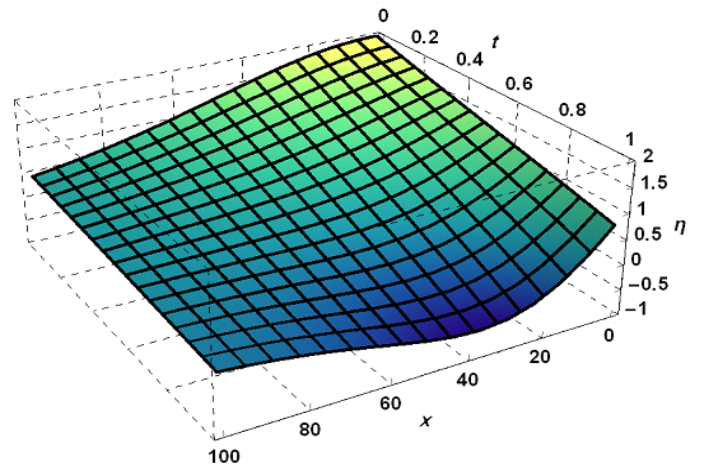


Figure 10. HAM solution of $\eta(x, t)$ for moderate-slope shore

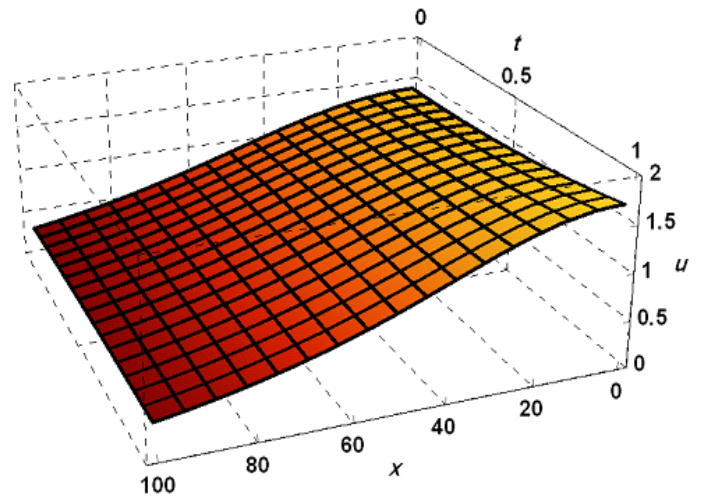


Figure 11. HAM solution of $u(x, t)$ for moderate-slope shore

3.3. Sharp-Slope Shores

For modeling a sharp-slope shore, the below shore profile is firstly considered:

$$h(x) = 5x - 500 \quad (51)$$

Then, the elevation profile Eq. (54) and surface elevation Eq. (55) are obtained by the following analytical expression for HAM after initial aunts of $\eta_0(x, t)$ and $u_0(x, t)$ are determined by Eq. and Eq. (53):

$$\eta_0(x, t) = 2 \operatorname{sech}[0.0137x]^2 \quad (52)$$

$$u_0(x, t) = 1.40071 \operatorname{sech}[0.0137x]^2 \quad (53)$$

$$\begin{aligned} \eta_1(x, t) = & -0.1534 t (-45.6435 \hbar \operatorname{sech}^2[0.0137x] \\ & -125 \hbar \operatorname{sech}^2[0.0137x] \tanh[0.0137x] + 1.25 x \hbar \\ & \operatorname{sech}^2[0.0137x] \tanh[0.0137x] + \\ & \hbar \operatorname{sech}^4[0.0137x] \tanh[0.0137x]) \end{aligned} \quad (54)$$

$$\begin{aligned} u_1(x, t) = & -0.0537 t (10 \hbar \operatorname{sech}^2[0.0137x] \\ & \tanh[0.0137x] + \hbar \operatorname{sech}^4[0.0137x] \tanh[0.0137x]) \end{aligned} \quad (55)$$

The auxiliary parameter of $\hbar$ is considered equal to -1, and the series will be expanded up to 10th-order. After the series become converged in HAM, the equations are solved on the bases of VIM approach by the following analytical expressions Eq. (56) and Eq. (57):

$$\begin{aligned} u_2 = & 1.400714 \operatorname{sech}^2(0.013693x) + 2t \\ & (0.2686566 \sinh(0.013693x) \operatorname{sech}^3(0.013693x) \\ &) \times t^2 (-0.00367014 \operatorname{sech}^8(0.013693x) - 0.07407 \\ & \operatorname{sech}^6(0.013693x) + 3.9265 \operatorname{sech}^4(0.013693x) \\ & - 2.57643 \operatorname{sech}^2(0.013693x) - 1.883156 \\ & \operatorname{sech}^3(0.013696x) \sinh(0.013693x) + \\ & 0.0257644 x \operatorname{sech}^2(0.01363x) - 0.0386465 x \\ & \operatorname{sech}^4(0.013693x) \end{aligned} \quad (56)$$

$$\begin{aligned} \eta_2 = & 2 \operatorname{sech}^2(0.013693x) + t(0.15336 \\ & \sinh(0.013693x) \operatorname{sech}^5(0.013693x) \\ & - 7.00357 \operatorname{sech}^2(0.013693x) + 0.1918x \\ & \sinh(0.013693x) \operatorname{sech}^3(0.013693x) \\ & - 0.1918 \sinh(0.013693x) \operatorname{sech}^3(0.013693x) + \\ & t^2 [-0.0101545 \operatorname{sech}^8(0.013693x) + 1.81583 \\ & \operatorname{sech}^6(0.013693x) - 4.076 \operatorname{sech}^4(0.013693x) \\ & - 1.861 \operatorname{sech}^2(0.013693x) + 0.03675x \\ & \operatorname{sech}^2(0.013693x) - 1.34325 \operatorname{sech}^3(0.013693x) \\ & \sinh(0.013693x) - 0.5373x \operatorname{sech}^5(0.013693x) \\ & \sinh(0.013693x) - 0.018393x \operatorname{sech}^6(0.013693x) \\ & - 0.040465x \operatorname{sech}^4(0.013693x) \end{aligned} \quad (57)$$

Then, the results are shown in figures 10-13. In figures 10, the graphical solutions of $\eta(x, t)$ and $u(x, t)$ are

represented. And also, the elevation and velocity profiles are illustrated versus time and position in figures 12 and 13. By comparing these figures, it is concluded that HAM and VIM solutions are similar.

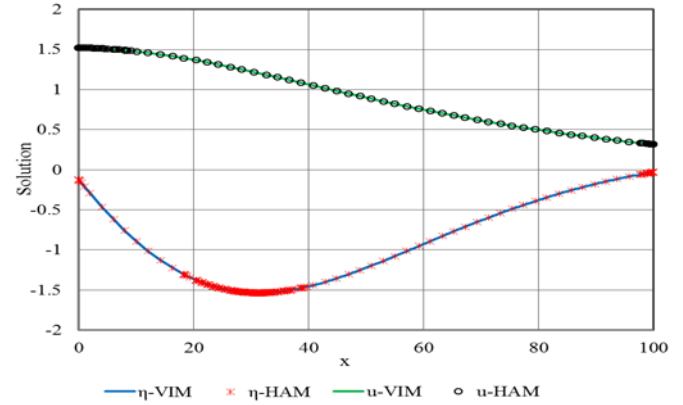


Figure 10. HAM and VIM solutions of $\eta(x, t)$ and $u(x, t)$ for sharp-slope shore at $t = 0.3$ in $0 \leq x \leq 100$

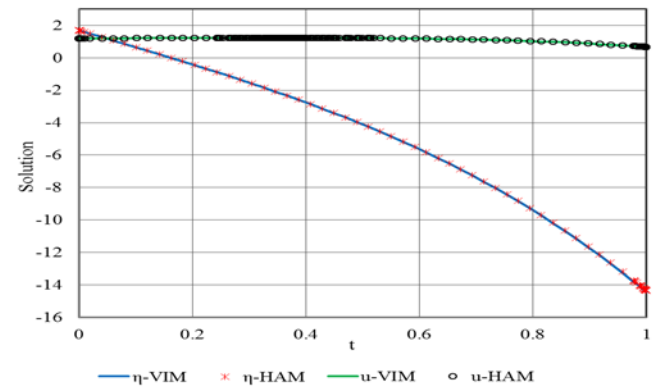


Figure 11. HAM and VIM solutions of $\eta(x, t)$ and $u(x, t)$ for sharp-slope shore at in $0 \leq t \leq 1$.

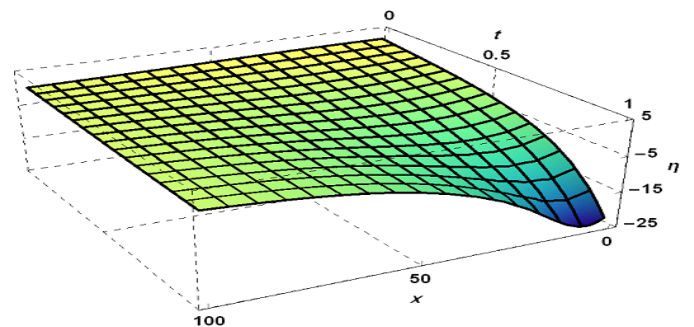


Figure 12. HAM solution of $\eta(x, t)$ for sharp-slope shore

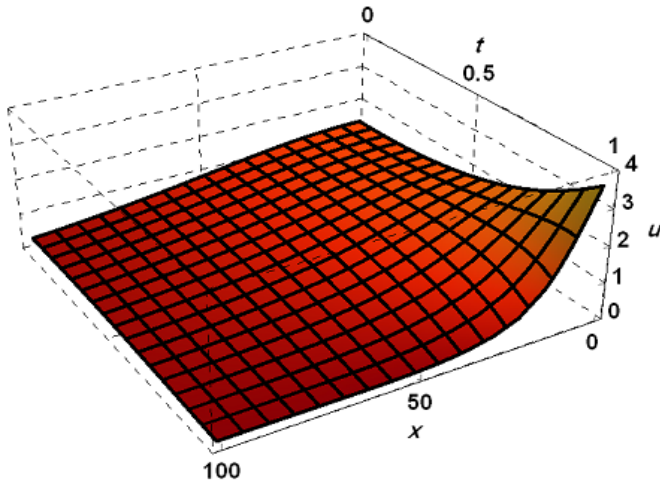


Figure 13. HAM solution of $u(x, t)$ for sharp-slope shore

4. Model verification

To verify the numerical models, two aforementioned slopes (semi-flat and moderate-slope shore), as examples, are selected and modeled in Flow-3D, and $\eta(x, t)$ s are shown in figures 16-17 at $t=0.3$ s for them. To compare the software and numerical results to the numerical results, correlation coefficients for each slope are obtained by Eq. (58) and shown in figures 17-19.

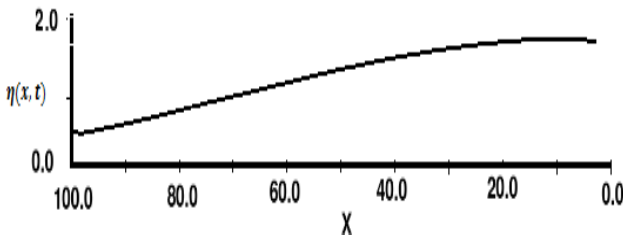


Figure 16. Flow-3D solutions of $\eta(x, t)$ for semi-flat shore at $t = 0.3$ in $0 \leq x \leq 100$

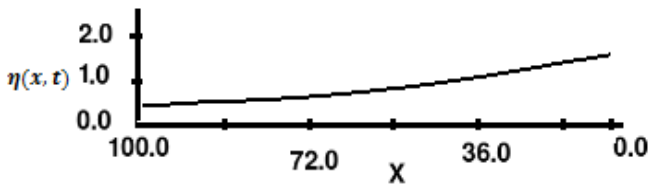


Figure 17. Flow-3D solutions of $\eta(x, t)$ for moderate-slope shore at $t = 0.3$ and $0 \leq x \leq 100$

$$CC = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{(\sum (x_i - \bar{x})^2)(\sum (y_i - \bar{y})^2)}} \quad (58)$$

Where x denotes the measured values from the numerical model and y is the measured values from flow 3D model. The value of $cc=1$ shows that two variables are exactly the same.

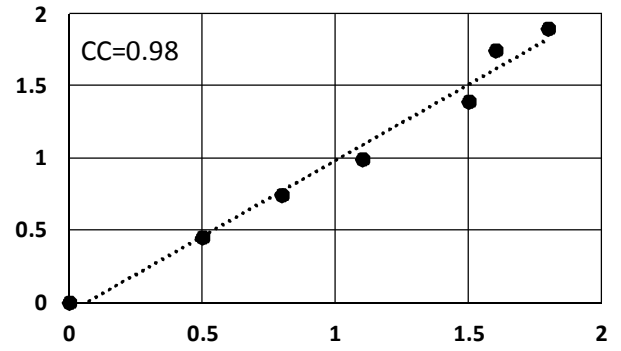


Figure 18. coefficient correlation of $\eta(x, t)$ between Flow-3D and numerical model results for semi-flat shore at $t = 0.3$ in $0 \leq x \leq 100$.

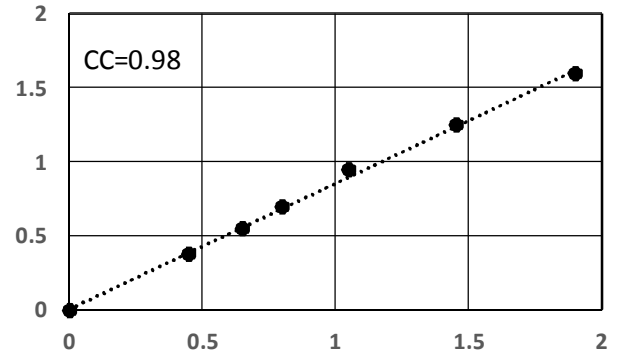


Figure 19. coefficient correlation of $\eta(x, t)$ between Flow-3D and numerical model results for moderate slope shore at $t = 0.3$ in $0 \leq x \leq 100$.

Conclusions

In this paper, the Homotopy Analysis Method (HAM) is applied to show the efficiency of this solution method in solving nonlinear surface wave propagation equations in shallow water. For this purpose, three shore profiles including semi-flat, moderate-slope, and sharp-slope shores are modeled by HAM and compared to the results obtained by Variational Iteration Method (VIM). The comparison represents that the results of both methods well coincide. However, it is remarkable to mention that HAM approach never discretizes, and using HAM rather than VIM provides a convenient solving method to control the convergence of approximation series, which is shown and observed during the solution process. Furthermore, as it is represented in the procedure of HAM, this method is more abridged and is not affected by computation round off errors. Ultimately, since the advantage of HAM outweighs VIM, HAM is highly recommended for solving the governing equation in shallow water in order to hind cast surface elevation and velocity of tsunami events at desired time and location.

6. References

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