obtained in the present model, could be related to the special finite difference scheme used here. However, the main reason of overestimation of C_L needs more investigations in future works.

(a)

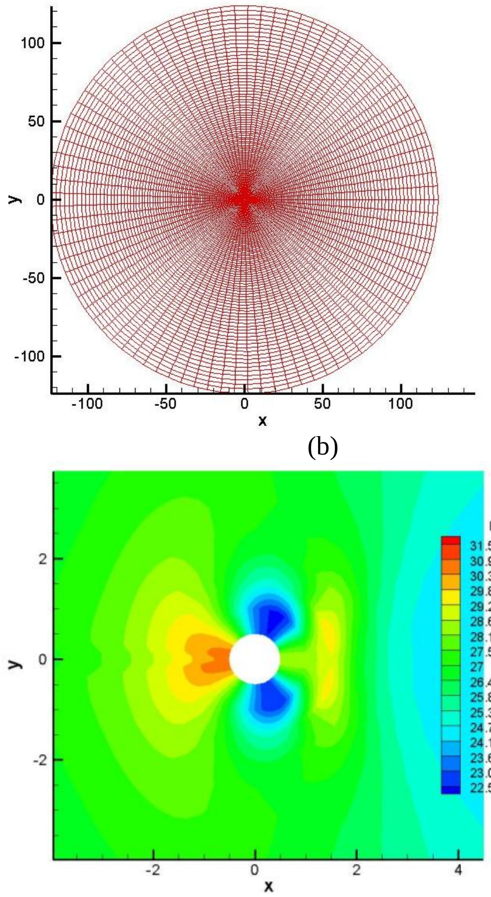

Fig. 2 (a): Computational grid in the physical domain. (b): Pressure contours around a circular cylinder for Re=100;

The present model, failed to improve the computation time. The time required for computation of eigenvalues of Jacobin matrices in the Steger–Warming formulation, may be the main reason for the high computational time.

Figure 3 shows the variation of lift force as a function of time at *Re=100*. As can be seen, numerical modeling of the phenomenon is able to present the real picture of the flow physics, since the oscillation in lift force is caused by the variation of pressure in effect of alternating vortex shedding in the wake of the riser. Fig. 4 shows the alternating nature of the vortices in four distinct time steps at *Re=100*. The well-known vortex street of Karman is quite obvious at this figure.

Table 2. Comparison between results obtained by the present work and other numerical and experimental data

Reference	Trriton [15] (dt=0.005)	Herfjord [14]	J.B. Wanderley [7]	Present study (dt=0.004)
Model type	experimental	Finite element	Finite difference (Beam-Warming)	Finite difference (Steger - Warming)
Reynolds no	100	100	100	100
Lift coefficient	0.34	0.34	0.313	0.37

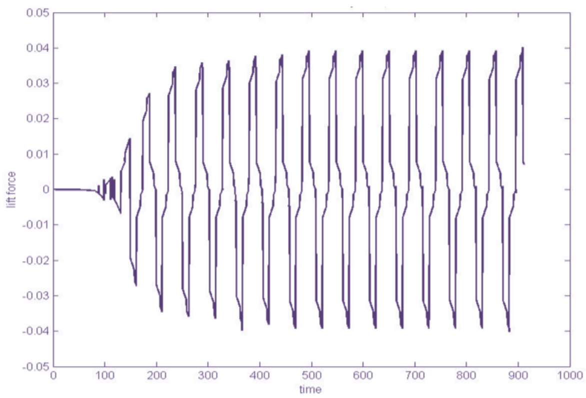

Fig. 3 Time variation of lift force at Re=100

After the accuracy of the model is verified at *Re=100*, the modeling was generalized to other Reynolds numbers. The Reynolds numbers used in the present study are in the range of 500 to 10000 which are proportional to the reduced velocity in the range of 0.5 to 10. The mass and damper coefficients used here, were $C\mu=1.88$ and $\zeta=5.42\times10\text{-}3$, which are the same coefficients used in [7] and [8]. Fig. 5 shows results obtained for the transversal displacement of the vibrating circular cylinder as a function of time. The time is traced in two states, one in the build-up of the upper branch and another at the top of the upper branch Khalak and Williamson [8]. Two values of reduced velocities of *Ur=4.5* in Fig. 5a and *Ur=6.5* in Fig. 5b correspond to the build-up and top part of the upper branch, respectively. The main reason that the lower branch in fig. 1 was not captured at present model may be related to the 2-D modeling using RANS equations, while the lower branch has been obtained in experimental model of Khalak and Williamson [8] which is 3-D.

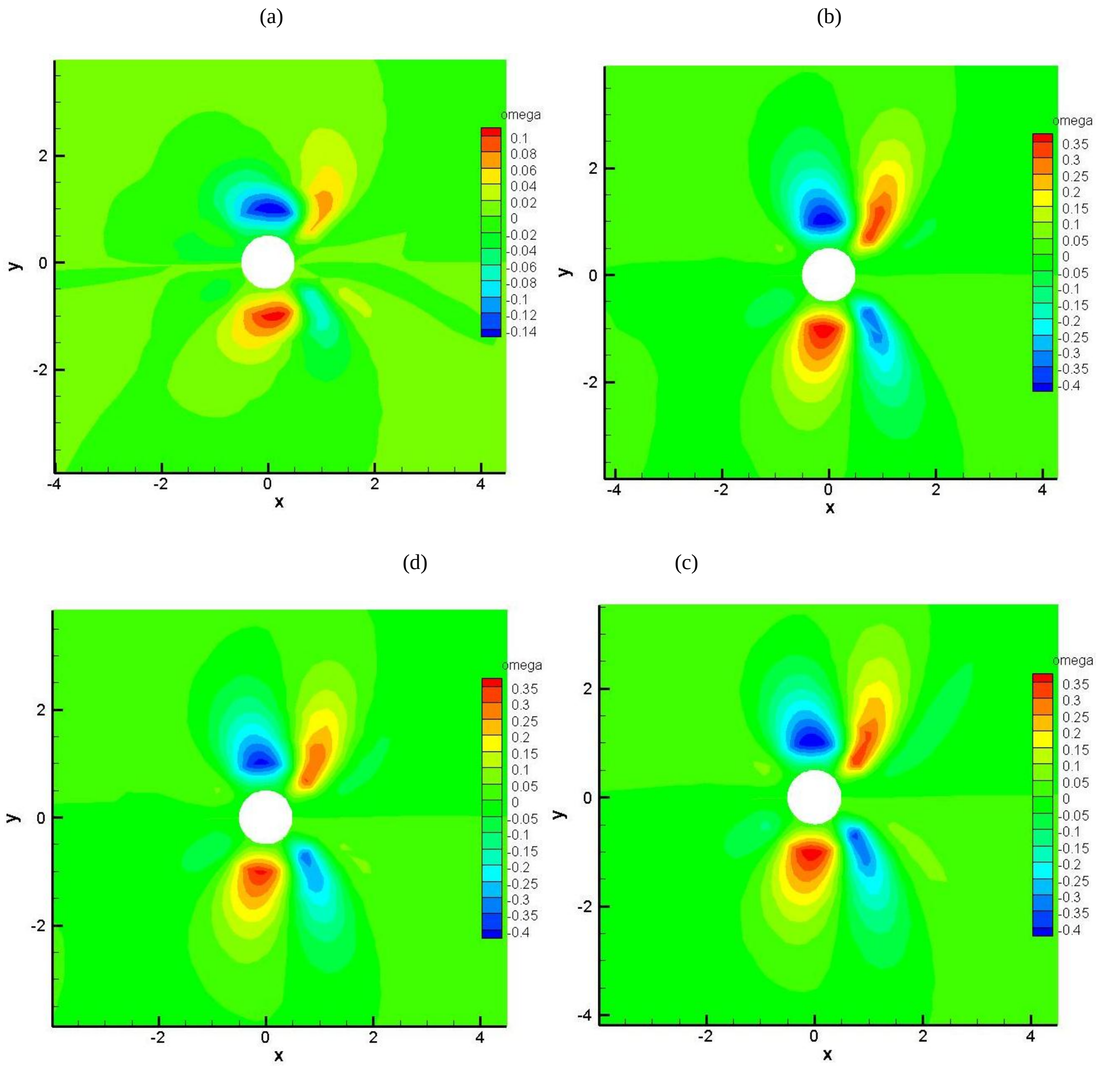

Fig. 4 Vortex strength contours at Re=100 in four time steps: (a) 100, (b) 500, (c) 1000, (d) 5000

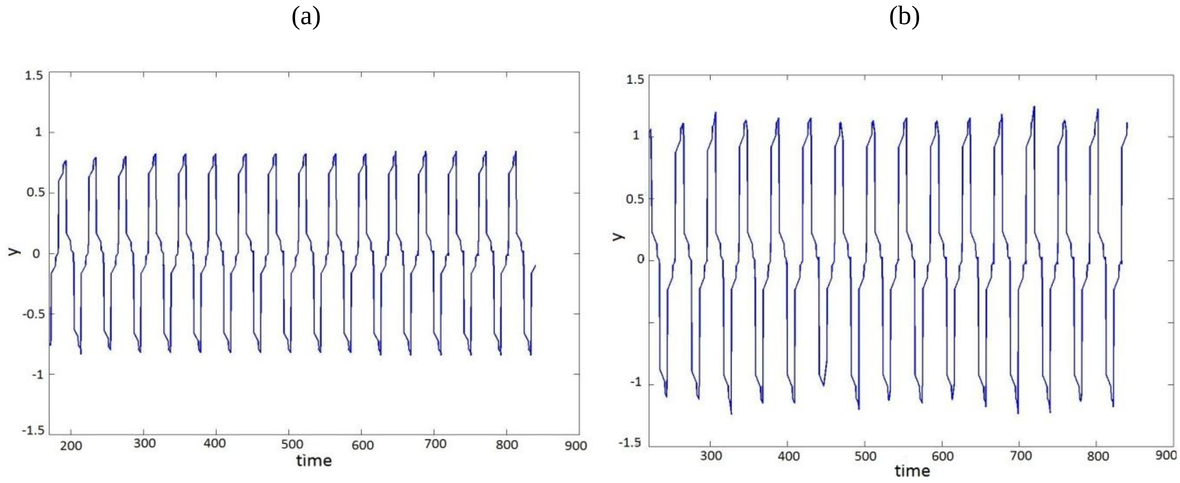

Fig. 5 Transversal displacement of the riser for (a) Ur=4.5; (b) Ur=6.5.

The main reason the displacement is greater for U_r=6.5 than U_r=4.5 is that U_r=6.5 is in the lock–in region, i.e. the region of top part of upper branch in Figure 1, where the synchronization of frequencies occurs. Figure 6 shows the spectral analysis of displacements for U_r=6.5. As seen, the main component of period of displacements is lied on T=39.81s which justify entering riser to lock–in area at that reduced velocity, since this time period is very close to the natural period of riser displacement (T_n=40s). The features presented here are in close agreement with those presented in Khalak, and Williamson [8].

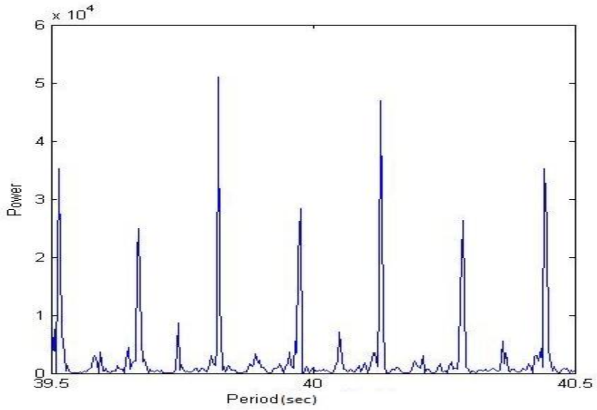

Fig. 6 Spectral analysis of riser displacement, U_r=6.5 (*Re*=6500)

5. Conclusions

Vortex-Induced vibration is a major concern in offshore oil and gas industry. This phenomenon can cause considerable damage to production lines. Here, a numerical code was developed to capture the vibration ranges of the risers and the comparison of the results show that the finite difference method can be considered as an efficient model tool to simulation of riser dynamics. Is is also indicated that the regime of so called the riser wake can also be captured using the present finite difference method.

First, the results were obtained at *Re=100,* and compared by data available in the literature. Then the solution was generalized to other *Re* (or *Ur*) values. The present mathematical model and respective numerical formulation were able to capture the upper branch of the amplitudes of oscillation reported in Khalak, and Williamson [8]. The lower branch obtained in the present investigation did not agree with the experimental data. To modeling this part of riser oscillation, it is needed to more robust models such 3-D ones to obtain an accurate results for lower branch regime.

The main features of present numerical investigation are:

- Using the k-ε turbulent model which is more suitable for separated flows.
- Formulating the conservative form of governing equations
- Applying the approximate factorization to increase the efficiency of solution.

Further investigations can include studying the problem with other turbulence models in CFD. Using the three-dimensional simulation and considering the more complicated geometries of the riser using overlapping Chimera Multi-block grids Houzeaux and Codina [13].

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